Midterm 2 Review

Note: This is not intended to be your only review. Please consult lecture and discussion notes, homework problems, worksheets, and quizzes.

1. Evaluate $\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy$.

2. Find the area of the region inside the circle $r = 2\cos(\theta)$ and outside the circle $r = 1$.

3. Find the volume of the solid that is under the surface $z = x^2 y$ and above the triangle in the xy-plane with vertices $(1, 0), (2, 1)$ and $(4, 1)$.

4. A lamina occupies the part of the disk $x^2 + y^2 \leq a^2$ that lies in the first quadrant. (Feel free to refer to formulas in section 15.3)

   (a) Find the centroid of the lamina.

   (b) Find the center of mass of the lamina if the density function is $\rho(x, y) = xy^2$.

5. Find the volume of the solid that is bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$.

6. Evaluate $\iiint_E xz dV$, where E is below the xy-plane, inside the sphere $x^2 + y^2 + z^2 = 4$ and the cone $3z^2 = x^2 + y^2$, and has $x \leq 0$.
   Hint: The cone makes an angle of $\frac{\pi}{3}$ with the negative z-axis.

7. Find the area of the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 1$ and $z = 2$.
   Hint: The area of a surface $z = f(x, y)$, $(x, y) \in D$, if f_x, f_y are continuous is $\iint_D \sqrt{(f_x(x, y))^2 + (f_y(x, y))^2 + 1} dA$

8. Evaluate $\iint_R (x+y)e^{x^2+y^2} dA$, where A is the rectangle enclosed by the lines $x-y = 0$, $x - y = 2$, $x + y = 0$, and $x + y = 3$.

9. Evaluate the line integral $\int_C \mathbf{F} \cdot dr$, where $\mathbf{F}(x, y, z) = (x + y^2)\mathbf{i} + xz\mathbf{j} + (y + z)\mathbf{k}$ and C is given by $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} - 2t\mathbf{k}, 0 \leq t \leq 1$.