Midterm 3 Review

Note: This is not intended to be your only review. Please also refer to notes from lecture and discussion, homework, and quizzes.

1. Consider the vector field $F(x, y, z) = (x^2e^z, x^2y, y^2)$. Compute the quantity:

$$\text{div}((1, z, 0) \times \text{curl}(F))$$

2. Evaluate the line integral

$$\int_C (-xy)dx + (xy)dy$$

where C is the boundary of the square $0 \leq x \leq 1, 0 \leq y \leq 1$, oriented counterclockwise.

3. Find a parametrization of the surface $x = 3z^2 + 8yz$ and use it to find the tangent plane at $(3, 0, 1)$.

4. Let C be the perimeter of the triangle in $\mathbb{R}^3$ with vertices $(1, 0, 3)$, $(2, 0, 3)$, and $(1, 1, 3)$ in order. If $F(x, y, z) = (xe^x, y^3 + x^2, xz)$, evaluate the line integral

$$\int_C F \cdot d\mathbf{s}.$$

5. Consider the vector field

$$\mathbf{F}(x, y, z) = (y^3 + z\cos(x), 3xy^2 + 1, \sin(x)).$$

   a) Is there a function $f(x, y, z)$ with $\nabla f = \mathbf{F}$? If so, find f. If not, explain why not.

   b) Compute $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is the line segment from $(0, 0, 1)$ to $(0, 2, 3)$.

6. Give a parametrization for the cone $x^2 + y^2 = z^2$ between $z = 0$ and $z = 2$. Use a surface integral to verify that the surface area is $4\sqrt{2}\pi$.

7. Suppose we have a fluid in $\mathbb{R}^3$ with flow given by the vector field $F(x, y, z) = (x, y, 1)$. Suppose we have a surface parametrized by $\phi(u, v) = (u + v, v, uv)$ for $0 \leq u \leq 1$, $2 \leq v \leq 3$. Find the flux through this surface. Assume the surface is oriented upwards.

8. Evaluate $\iint_S F \cdot dS$ where $F = (2xz, 1 - 4xy^2, 2z - z^2)$ and S is the boundary of the solid bounded by $z = 6 - 2x^2 - 2y^2$ and the plane $z = 0$.