Math 2263 Fall 2019 - Quiz 7
10/31/19 - 20 Minutes

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There are no calculators allowed on this quiz. Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

1 (5 pts). Use polar coordinates to find the y-coordinate of the center of mass of the semi-circular region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ if the density is $\rho(x, y) = y$.

$$\bar{y} = \frac{\iint_B y\rho \, dA}{\iint_B \rho \, dA}. \quad \text{Have } B_{r\theta} = \left\{ \begin{matrix} 0 \le r \le 3 \\ 0 \le \theta \le \pi \end{matrix} \right\} \text{ (semicircle).}$$

$\boxed{y = r\sin\theta}$

Have $\rho = y$. Go directly to polar; get

$$\bar{y} = \frac{\int_0^3 \int_0^\pi (r\sin\theta)^2 \, \underline{r} \, d\theta \, dr}{\int_0^3 \int_0^\pi (r\sin\theta) \, \underline{r} \, d\theta \, dr} = \frac{\int_0^3 \int_0^\pi r^3 \sin^2\theta \, d\theta \, dr}{\int_0^3 \int_0^\pi r^2 \sin\theta \, d\theta \, dr}$$

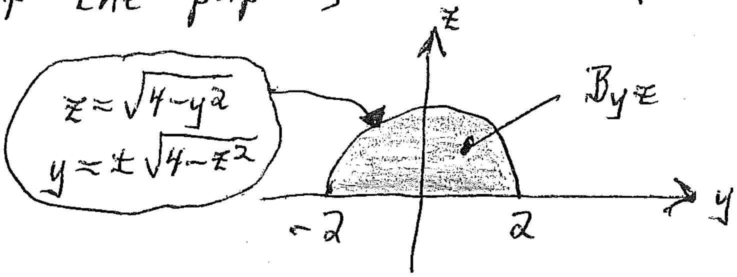
$$\text{Top} = \int_0^3 r^3 \, dr \cdot \int_0^\pi \frac{1 - \cos(2\theta)}{2} \, d\theta = \frac{3^4}{4} \cdot \frac{\pi}{2}$$

$$\text{Bot} = \int_0^3 r^2 \, dr \cdot \int_0^\pi \sin\theta \, d\theta = \frac{3^3}{3} (-1) \left[\cos\theta\right]_0^\pi = \frac{3^3}{3} \cdot 2$$

$$\frac{\text{Top}}{\text{Bot}} = \frac{3^4 \pi}{8} \frac{1}{3^2 \cdot 2} = \frac{3^2 \pi}{16} = \frac{9\pi}{16} = \bar{y}$$

2 (5 pts). Let E be the solid region bounded by the cylindrical surface $z = \sqrt{4-y^2}$ and the planes $z = 0$, $x = 1$, $x + y = 4$. Set up the triple integral $\iiint_E f(x,y,z)\,dV$ as an iterated integral with base B in the yz-plane using *two* different orders; i.e., $dx\,dy\,dz$ and $dx\,dz\,dy$. Indicate the limits of integration clearly. (Use cartesian coordinates only.)

Cylinder $z = \sqrt{4-y^2}$ means $z^2+y^2=4$ and $z \geq 0$.
E is bounded by $z=0$, the cylinder, $x=1$, and $x = 4-y$. We put sun at $x=+\infty$ and then send light rays into E. With x out of the paper, we see (from $x = 10000$ miles)



ray enters at $x = 4-y$
ray exits at $x = 1$

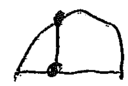
Note $4-y > 1$ since $3 > y$ (look at B_{yz}).

so, $E = \left\{ \frac{(y,z) \text{ in } B_{yz}}{1 \leq x \leq 4-y} \right\}$. To finish, use type 1 and type 2 description for B_{yz}. Get:

$$E = \left\{ \begin{array}{c} -2 \leq y \leq 2 \\ 0 \leq z \leq \sqrt{4-y^2} \\ \hline 1 \leq x \leq 4-y \end{array} \right\} \quad \text{and} \quad \left\{ \begin{array}{c} 0 \leq z \leq 2 \\ -\sqrt{4-z^2} \leq y \leq \sqrt{4-z^2} \\ \hline 1 \leq x \leq 4-y \end{array} \right\}$$

hence

$$\text{integral} = \int_{-2}^{2} \int_0^{\sqrt{4-y^2}} \int_1^{4-y} f\,dx\,dz\,dy$$



$$\text{integral} = \int_0^2 \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} \int_1^{4-y} f\,dx\,dy\,dz$$

(For fun, take $f = 1$ and <u>calculate</u> both integrals.)