READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 11 pages, including this cover page. Check to see if any are missing.
PRINT on the upper right-hand corner all the requested information, and sign your name.
Put your initials on the top of every page, in case the pages become separated.
Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 12 machine-graded problems worth 12 points each and 6 hand-graded problems
worth 156 points together for a total of 300 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-12):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the
answer sheet, and carefully enter all the requested information according to the instruc-
tions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET.
When you have decided on a correct answer to a given question, circle the answer in this
booklet and blacken completely the corresponding circle in the answer sheet. If you erase
something, do so completely. Each question has a correct answer. If you give two different
answers, the question will be marked wrong.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 13-18):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the
School of Mathematics may for any reason request a regrade of the machine graded part.
All regrades will be based on responses in the test booklet, and not on the machine graded
response sheet. Any problem for which the answer is not indicated in the test booklet, or
which has no relevant accompanying calculations will be marked wrong on the regrade.
Therefore work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet
between two pages of this booklet (make a sandwich), with the side marked “GENERAL
PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning
in your exam.

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1. \(\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1} = \)

A. -3
B. 3
C. \(\frac{1}{3}\)
D. 1
E. Does not exist

2. The derivative of \(f(x) = e^x \ln x + (x^2 + 1)^4\) is

A. \(e^x \ln x + 16x^4\)
B. \(e^x (\frac{1}{x}) + 4(x^2 + 1)^3\)
C. \(e^x (1 + \frac{1}{x}) + 4(x^2 + 1)^3\)
D. \(e^x (\ln x + \frac{1}{x}) + 8x(x^2 + 1)^3\)
E. \(e^x \ln x + 8x(x^2 + 1)\)

3. The definite integral \(\int_0^1 (e^{2x} + \sqrt{x}) \, dx\) is

A. \(\frac{1}{2}e^2 + \frac{1}{6}\)
B. \(\frac{1}{2}e^2 + \frac{2}{3}\)
C. \(e^2 + \frac{1}{6}\)
D. \(e^2 + \frac{2}{3}\)
E. \(\frac{1}{2}e^2 - \frac{1}{2}\)
4. \[ \lim_{x \to \infty} \frac{2x^2 + 3}{5x^2 + 2x + 1} = \]

A. \( \frac{2}{5} \)

B. \( \frac{3}{2} \)

C. 1

D. \( \infty \)

E. Does not exist

5. The equation \( \frac{1}{x} + \frac{2}{y} = 2 \) defines \( y \) implicitly as a function of \( x \). Find the tangent line to the curve \( \frac{1}{x} + \frac{2}{y} = 2 \) at \((1, 2)\).

A. \( y - 2 = \frac{1}{4}(x - 1) \)

B. \( y - 1 = \frac{1}{4}(x - 2) \)

C. \( y - 2 = 0 \)

D. \( y - 2 = -2(x - 1) \)

E. \( y - 2 = \ln 2(x - 1) \)

6. \( \int_0^2 5xe^x \, dx = \)

A. \( 5e^2 \)

B. \( 5e^2 + 1 \)

C. \( 5e^2 + 5 \)

D. \( 5e \)

E. \( 5(e - 1) \)
7. The improper integral \( \int_{1}^{\infty} \frac{1}{(2x+1)^3} \, dx \)

A. converges and has value \( \frac{1}{4} \)
B. converges and has value \( \frac{1}{25} \)
C. converges and has value \( \frac{1}{35} \)
D. converges and has value 0
E. diverges

8. An object moving along a line has velocity \( v(t) = 3t^2 + 2t + 1 \) meters per minute. How far does the object travel during the first 3 minutes?

A. 16 meters
B. 20 meters
C. 25 meters
D. 34 meters
E. 39 meters

9. \( f(x, y) = \frac{3xy+2}{2x+5y} \). Then \( \frac{\partial f}{\partial x} \) evaluated at \( x = 1, y = 1 \) is

A. \( \frac{26}{49} \)
B. \( \frac{11}{49} \)
C. \( \frac{16}{7} \)
D. \( \frac{5}{7} \)
E. \( \frac{3}{2} \)
10. \( \frac{dy}{dx} = \frac{1}{2} e^{x-y} \). If \( y(0) = 0 \), what is \( y(1) \)?

A. 1
B. e
C. \( \frac{e}{2} \)
D. \( \ln(e + 1) \)
E. \( \ln \frac{e+1}{2} \)

11. \( \int_{0}^{1} (x + 1)(x^2 + 2x - 3)^4 \, dx = \)

A. \( \frac{243}{10} \)
B. \( -\frac{243}{10} \)
C. \( \frac{243}{5} \)
D. \( -\frac{243}{5} \)
E. \( \frac{32}{5} \)

12. The amount of a certain radioactive substance remaining after \( t \) years is given by \( Q(t) = Q_0 e^{kt} \) grams, where \( Q_0 \) and \( k \) are two constants (depending on the substance). Suppose there are 100 grams of the substance initially and 50 grams remain after 10 years. Find \( Q_0 \) and \( k \).

A. \( Q_0 = 50, \ k = -\ln 10 \)
B. \( Q_0 = 50, \ k = -\ln 2 \)
C. \( Q_0 = 100, \ k = -\frac{1}{10} \ln 2 \)
D. \( Q_0 = 100, \ k = -\ln 10 \)
E. None of the above
13. (25 pts) Find the absolute maximum and absolute minimum value of $f(x) = x^3 + 2x^2 + 10$ for $-3 \leq x \leq 2$. 
14. (25 pts) Use the trapezoidal rule with $n = 4$ to approximate the definite integral

$$\int_{0}^{4} \frac{1}{x^2 + 4} \, dx.$$  

(The trapezoidal rule: $\int_{a}^{b} f(x) \, dx \approx \frac{\Delta x}{2} [f(x_1) + 2f(x_2) + \cdots + 2f(x_n) + f(x_{n+1})]$, where $\Delta x = \frac{b-a}{n}$ and $x_i = a + (i-1)\Delta x$)
15. (25 pts) Find all critical points for the function \( f(x, y) = x^3 - 3xy + y^3 \) and classify each as a relative maximum, a relative minimum or a saddle point.
16. (25 pts) Find the area of the region bounded by the line $y = x + 2$ and the curve $y = x^2 - 4$. 
17. (1) (15 pts) For \( f(x) = x^4 - 2x^2 + 3 \), find the intervals on which \( f(x) \) is increasing or decreasing. Also indicate where \( f(x) \) has a relative maximum or minimum.

(2) (16 pts) For \( f(x) = xe^{2x} \), find the intervals on which \( f(x) \) is concave upward or concave downward. Also indicate where \( f(x) \) has an inflection point.
18. (25 pts) A farmer with 240 feet of fencing wants to enclose a rectangular field. A building is on one side of the field so no fence is needed on this side. Determine the dimensions of the field that will enclose the largest area. Please explain why your answer gives the largest area.