Math 1142  Final Exam

December 14, 2006

Last name: ___________________ First name: ___________________
ID: __________________________ TA: __________________________
Recitation time: ______________

I certify that the answers on this exam are my own, produced in accordance with all University and Institute of Technology policies on Scholastic Conduct. Signature: __________________________

Instructions:

• There are 13 problems on this exam.
• You have 3 hours to complete the exam.
• There are 260 points available; the worth of each problem is indicated in brackets.
• You are not allowed to have any sort of books, notes, cell phones, calculators, or computers outside of your backpack in the exam room.
• You must show all steps in your solutions and make your reasoning clear with English sentences to earn credit.
• If you need more space, there are blank pages at the end of the exam.
• Please clearly mark your final answer for each problem.

Test tips:

• You may want to read through all of the questions first, then start with the questions that you feel the most comfortable with and save the more difficult questions for the end.
• If you're stuck on a question, you may want to move on and come back to it at the end.

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1. (a) [15 pts] Find the equation of the line tangent to the graph of 
\( f(x) = x\sqrt{x^2 + 16} \) at \( x = -3 \).

(b) [5 pts] Is the function increasing or decreasing at this point? Why or why not?
2. Let \( f(x) = \frac{x^2 - 14x + 40}{x^3 - 2x^2 - 8x} \)

(a) [5 pts] What is the domain of \( f(x) \)?

(b) [5 pts] Compute \( \lim_{x \to 1} f(x) \).

(c) [5 pts] Compute \( \lim_{x \to 4} f(x) \).

(d) [5 pts] Compute \( \lim_{x \to \infty} f(x) \).
3. Find \( \frac{dy}{dx} \) if:

(a) [4 pts] \( y = x^2 + \ln(x^2) \).

(b) [4 pts] \( y = (e^x + \frac{5}{x})^{20} \).

(c) [4 pts] \( y = \frac{x^2 + 1}{x^2 - 1} \).
(d) [4 pts] \( y = \log_9(81^x) \).

(e) [4 pts] \( x^4 - y^4 = -5xy \).
4. Consider the graph of the function $f(x)$ below.

(a) [5 pts] At which points is $f(x)$ discontinuous?

(b) [5 pts] What is $f(5)$? What is $\lim_{x \to 5} f(x)$?

(c) [5 pts] Is $f'(10)$ positive or negative? How can you tell?

(d) [5 pts] On which intervals is $f''(x)$ positive? How can you tell?
5. [20 pts] Find the absolute maximum and minimum of $f(x) = (x - 2)^{2/3}$ on the interval $[1, 10]$. (HINT: $(8)^{1/3} = 2$.)
6. [20 pts] Find the location of all relative maxima and minima for the function 
\[ y = 3x^4 - 16x^3 + 18x^2 - 10. \]
7. [20 pts] A box with a square base is to be constructed. The box needs to hold 100 cubic meters. The material for the bottom of the box costs $20 per square meter while the sides cost $5 per square meter. The box has no top. How should the box be built so that it costs as little as possible?
8. Find the following integrals.

(a) [5 pts] \( \int \sqrt{x} + \frac{1}{x^3} \, dx \)

(b) [5 pts] \( \int \frac{4}{x} + e^{-2x} \, dx \)
(c) [5 pts] \( \int \frac{x^3}{x^4 + 1} \, dx \)

(d) [5 pts] \( \int xe^{2x} \, dx \)
9. (a) [10 pts] Find the area under the curve \( f(x) = 2x^2 + 3 \) over the interval \( 0 \leq x \leq 1 \).

(b) [10 pts] Find the average value of the function \( f(x) = 2x^3 + x + 1 \) on the interval \( 0 \leq x \leq 2 \).
10. A delivery truck leaves a warehouse at 6 a.m. After $t$ hours, the truck’s velocity is $v(t) = 60 - 60e^{-t}$.

(a) [10 pts] What is the truck’s acceleration as a function of time? (i.e. what is $a(t)$?)

(b) [10 pts] How far does the truck travel between 6 a.m. and 10 a.m.? 
*You may leave your answer in terms of $e$.**
11. [20 pts] Find the particular solution to the differential equation \( \frac{dy}{dx} = \frac{e^{3x}}{y^2} \)
satisfying \( y = 0 \) when \( x = 0 \). *Please solve your final answer for y.*
12. The useful life of a type of computer is measured by a random variable $X$ with probability density function

$$f(x) = \begin{cases} 
.2e^{-2x} & \text{if } x \geq 0 \\
0 & \text{if } x < 0 
\end{cases}$$

where $x$ denotes the number of years a randomly selected computer has been in use.

(a) [10 points] What is the probability that a randomly selected computer will last at least 5 years? You may leave your answer in terms of $e$.

(b) [10 points] What is the probability that a randomly selected computer will last 1 year or less? You may leave your answer in terms of $e$. 
13. Part 1: Let \( f(x, y) = x^3 + x^2y + xy + y^5 + 2 \).

(a) [5 pts] Find the partial derivative \( f_x(x, y) \).

(b) [5 pts] Find the partial derivative \( f_y(x, y) \).
Part 2: Let $f(x, y) = ye^x$.

(c) [5 pts] Find the partial derivative $f_x(x, y)$.

(d) [5 pts] Find the partial derivative $f_y(x, y)$. 