Math 1142 Quiz 2 Solutions

1. (4 points) There are two points of the graph of \( y = x^3 \) where the tangent lines are parallel to \( y = x \). Find these two points.

Solution: Finding the points on this graph where the tangent lines are parallel to \( y = x \) is the same as finding the points where the tangent lines have slope equal to the slope of the line \( y = x \), which is 1. This is the same as finding the points where the derivative (of \( x^3 \)) is 1. The derivative of \( x^3 \) is \( 3x^2 \), and so the x-coordinates of the points we’re asked to find are the solutions to the equation

\[
3x^2 = 1
\]

These are

\[
x = \pm \sqrt{\frac{1}{3}}
\]

and so the points on the graph of \( y = x^3 \) that we’re looking for are

\[
\left( \sqrt{\frac{1}{3}}, \left( \sqrt{\frac{1}{3}} \right)^3 \right) \quad \text{and} \quad \left( -\sqrt{\frac{1}{3}}, \left( -\sqrt{\frac{1}{3}} \right)^3 \right)
\]

2. (6 points; 3 each) Determine if the following limits exist. If they exist, compute their limit.

(a) \( \lim_{x \to 3} \frac{x}{x - 3} \)

(b) \( \lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} \)

Solution:

(a) As \( x \) approaches 3, the numerator, \( x \), approaches 3 while the denominator, \( x - 3 \), approaches 0. Therefore the limit does not exist.

(b) Applying limit laws alone won’t be enough to solve this problem (you’ll end up with \( \frac{0}{0} \), which is meaningless), so, before trying to compute the limit, something must be done to simplify the fraction that we’re given. Notice that the quadratic in the numerator factors as

\[
x^2 - x - 6 = (x - 3)(x + 2),
\]

which means that

\[
\frac{x^2 - x - 6}{x - 3} = \frac{(x - 3)(x + 2)}{x - 3} = x + 2
\]

Using this fact, we can now compute the limit:

\[
\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} = \lim_{x \to 3} x + 2 = 3 + 2 = 5
\]