Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) Match the following functions with their graphs. Give reasons for your choices.

(a) \( z = e^{y^2+x^2} \)
(b) \( z = \sin(x^2 - y^2) \)
(c) \( z = \sin(x + y) \)
(d) \( z = \frac{1}{1 + x^2 + y^2} \)
(e) \( z = e^{y^2-x^2} \)

Answers

I-e Because it is zero on both diagonals \( x = y \) and \( x = -y \) or because it gets big when \( x = 0 \) (1pt)
II-a Because it gets big on circles \( x^2 + y^2 = k \) as \( k \to \infty \) (1pt)
III-d Because it is big at \((0,0)\) and tends to 0 on points far away from the origin. (1pt)
IV-b Because it has the same value when \( x^2 - y^2 = 0 \), that is, when \( x = y \) and \( x = -y \). (1pt)
V-c Because it is constant on lines parallel to \( x = -y \) (1pt)

(2) Find the indicated partial derivatives:

\[ f(x, y) = (\sin x)^y \]

(a) \( f_x = y \cdot (\sin x)^{y-1} \cdot \cos x \) (1pt)

(b) \( f_y = (\sin x)^y \cdot \ln(\sin x) \) (2pt)

(c) \( f_{xy} = \cos x \cdot [y(\sin x)^{y-1}]' = \cos x[(\sin x)^y - 1 + y(\sin x)^{y-1} \ln(\sin x)] \) (2pt)