Assignment Fourteen

Geese

My math teacher Epstein
liked to call me to the blackboard.
He said that my head was good only for hats,
and that a bird with brains like mine
would fly backwards.
He sent me to tend the geese.

Now, at a distance of years from his sentence,
when I sit under the palm tree
with my three beautiful geese,
I think that math teacher of mine was farsighted.
He was right,
because nothing makes me happier
than to watch them now
falling upon bread crumbs,
joyful tails wagging,
or freezing for a moment
under beads of water
when I spray them
with a hose,
holding their heads erect,
bodies stretched back
as if remembering faraway lakes.

Since then my math teacher has died,
together with the math problems
I could never solve.
I like hats
and always at evening
when the birds return to the tree
I look for the one flying backwards.

-Agi Mishol, translated from the Hebrew by Lisa Katz

1. (Tuesday Workshop)

   (a) Use integration by parts to find an expression for \( \int \sin^n(x) \, dx \) in terms of \( \int \sin^{n-2}(x) \, dx \).
(b) Use this formula to quickly compute \( \int_{0}^{\pi/2} \sin^{10}(x) \, dx \) and \( \int_{0}^{\pi/2} \sin^{11}(x) \, dx \).

(c) Find a general formula for \( \int_{0}^{\pi/2} \sin^{2n}(x) \, dx \) and \( \int_{0}^{\pi/2} \sin^{2n+1}(x) \, dx \).

(d) What does the inequality

\[
\int_{0}^{\pi/2} \sin^{2n-1}(x) \, dx < \int_{0}^{\pi/2} \sin^{2n}(x) \, dx < \int_{0}^{\pi/2} \sin^{2n+1}(x) \, dx
\]

(1)

tell you about \( \pi \)? This computation led to Wallis’s product formula, which you can find in appendix 2 to chapter 10 of the textbook.

2. Section 10.3, problems 11, 18. Section 10.4, problems 5, 8, 9, 20, 33. Section 10.7, problems 1, 2, 7, 10, 11, 18, 24.

3. Section 10.8, problems 10, 19, 26, 45, 58, 81, 90. Check the answers to the even-numbered problems by differentiating.