Assignment Two

In mathematics the art of proposing a question must be held of higher value than solving it.  
-Georg Cantor

1. (Workshop Problems for this week) Solve the following problems by algebra alone—no calculus techniques allowed!

(a) Find the slope of the tangent line to $y = x^2$ at the point (3, 9). (This type of problem was originally solved by Archimedes.)

(b) Find the slope of the tangent line to $y = x^3 - 7x + 6$ at the point (0, 6).

(c) Find the slope of the tangent line to $y^2 = x^3 + x^2$ at the point (3, 6). (This type of problem was originally solved by Pierre de Fermat.)

(d) Redo these three problems, leaving the coordinates of the point of tangency, $(x_0, y_0)$, as variables.
2. Section 2.2, problems 1, 3, 4. Section 2.3, problems 1, 8, 23, 26, 28, 31, 33, 44. Section 2.4, problems 1, 6, 10, 12. Section 2.5, problems 15, 20a, 20c, 20f.

3. Make a list of 10 real world quantities and their derivatives, e.g. position/velocity. Try to come up with unusual and interesting examples.

4. The derivative of the area of a circle, $\pi r^2$, is the perimeter, $2\pi r$. This phenomenon is nicely explained in Simmons section 2.4, example 3d. For a square of side length $s$, the area is $s^2$ and the perimeter is $4s$—not quite the derivative. But if we think of area and perimeter as functions of $r = \frac{1}{2}s$, then they are $4r^2$ and $8r$ respectively, and again, perimeter is the derivative of area. Why is this $r$ the “right” variable to consider? Try to find similar formulas for an equilateral triangle and for a regular polygon in general.

5. Find the slope of the tangent line to $y = 2^x$ at $(0, 1)$, by approximation, to 3 digits of accuracy. Do the same for $y = 3^x$ and $y = \left(\frac{1}{2}\right)^x$. 