Assignment Nine

In mathematics you don’t understand things. You just get used to them.
- John Von Neumann

1. (Thursday Workshop/Practice Exam 1)
   (a) Let \( f(x) = \frac{x^2 + 3}{x+4} \). Find \( f'(x) \) and \( f''(x) \). Simplify your answers.
   (b) Find all the roots of \( f(x) \), \( f'(x) \), and \( f''(x) \). Find all the vertical asymptotes of these functions.
   (c) Use the information of part (b) to sketch a graph of \( f(x) \), labeling any x-intercepts, maxima/minima, inflection points, and vertical asymptotes.

2. (Thursday Workshop/Practice Exam 1) Sketch the graph of a continuous function \( f(x) \) with the following properties:
   • \( f'(x) > 0 \) for \(-3 < x < 0\), \( 0 < x < 1 \), \( 2 < x \);
   • \( f'(x) < 0 \) for \( x < -3 \), \( 1 < x < 2 \);
   • \( f''(x) > 0 \) for \( x < -2 \), \( 0 < x < 1 \), \( x > 1 \);
   • \( f''(x) < 0 \) for \(-2 < x < 0 \).
   Why do these conditions imply that the graph has a sharp corner at \( x = 1 \)?

3. (Thursday Workshop/Practice Exam 1) Compute the square root of 10 using Newton’s method with \( x_1 = 3 \) and three further iterations. Using the error bound \( e_{i+1} < \frac{|k e_i^2|}{f'(x_i)} \) and the fact that \( e_1 < \frac{1}{6} \), show that \( e_4 < \frac{1}{1,000,000,000} \).

4. (Thursday Workshop/Practice Exam 1) You want to build a box with a square base and an open top. The base material costs four dollars per square foot, and the sides cost seven dollars per square foot. If the box is to have volume 12 cubic feet, what dimensions minimize the cost? Show that that your answer is a minimum.

5. (Thursday Workshop/Practice Exam 1) Find the following limits:
   (a) \( \lim_{x \to 1} \frac{x^3 - 7x + 6}{\ln(x)} \)
   (b) \( \lim_{x \to \infty} \frac{x^2}{e^x} \)

6. (Practice Exam 2) Chapter 4 additional problems (pages 156-162) 13, 27, 65, 77, 119(c).


8. Find an approximate solution to \( x = \cos(x) \) with Newton’s method, using \( x_1 = 1 \) and three further iterations. (Note: every time your calculator evaluates a trig function and gets a decimal answer, you are using a different kind of approximation, one which we will study next semester. So we really have approximations within approximations in this problem.)