Assignment 3

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I was reading up on my Hellenic math: no zero yet, no transfinite set theory, no sine or cosine, just a Brotherhood that felt divine to its practitioners. You were back in California, riding on the waves, your father gone awhile, and me, you said, no consolation outside of the bed. I've never yet let go of zero, the multiplier best employed on incommensurate sums, either one on top a lovelier ratio, but when divided through becomes a number that can't compute but instead goes on, in proof, diminished by each digit that it spews.

Eudoxus did not quite anticipate you, a curve that skims but will not touch the bottom line. And then the Romans came, and all was lost for quite some time.

-Heather Green

1. Practice Exam for Tuesday Workshop

   (a) Find \( \int \frac{1}{x^2\sqrt{x^2+9}} \, dx \).

   (b) Explain why the following integral is improper, determine whether it diverges or converges, and if it converges, compute it: \( \int_{0}^{\pi/2} \frac{\cos(x)}{\sqrt{\sin(x)}} \, dx \).

   (c) Find \( \int e^{x^2} \cdot x^{11} \ln(x) \, dx \).

   (d) The integral \( \int_{0}^{\infty} \frac{1}{x\ln(x)} \, dx \) is improper for three reasons. Determine whether the area of this region is finite or infinite at each location where it extends to infinity.

   (e) Find the value of \( \int_{3}^{4} \frac{x^2+x+1}{x^2-3x+2} \, dx \).

2. Thursday Workshop (this project will be continued next week): As a full class, make a comprehensive list of ways to approximate \( \pi \). You might think about geometric quantities, about integrals known to equal \( \pi \), about Newton’s method, or about infinite sums or products equal to \( \pi \). Then, breaking into small groups, you will be assigned one approximation technique. Answer the following questions:

   (a) Use your technique to give five increasingly good approximations to \( \pi \).

   (b) How far can your method go? If possible, give a formula or an algorithm to find the “nth” approximation.

   (c) Is there an error bound for your method? Can you say how accurate each approximation is?

   (d) How many steps does your technique take to get two decimal digits of \( \pi \)? Five decimal digits? 100 decimal digits? Would you say it is practical for computing \( \pi \)?
3. Approximation by Trapezoids: From section 10.9 of the textbook, do problems 2, 3, 4, but use approximation by trapezoids instead of Simpson’s rule. Also, for 2 and 4, give an error bound for each approximation using the formula $\epsilon \leq \frac{M_2(b-a)^3}{12n^2}$. For 4, you might need a graphing calculator to find $M_2$. Finally, for each integral, how large would $n$ need to be to guarantee $\epsilon < \frac{1}{1,000,000}$?

4. Simpson’s Rule: From section 10.9 of the textbook, do problems 4, 5, 6. Also, for 4 and 6, give an error bound for each approximation using the formula $\epsilon \leq \frac{M_4(b-a)^5}{180n^4}$. For 4, you might need a graphing calculator to find $M_4$. Finally, for each integral, how large would $n$ need to be to guarantee $\epsilon < \frac{1}{1,000,000}$?

5. Miscellaneous challenge integrals: choose 2 of these.

   (a) $\int (\arcsin(x))^2\, dx$

   (b) $\int \frac{1}{1+e^x}\, dx$

   (c) $\int_{-\infty}^{\infty} \frac{1}{x^4+1}\, dx$

   (d) $\int \sqrt{\tan(x)}\, dx$