Assignment 4

*If six was nine*
*I don’t mind, I don’t mind.*
-If 6 Was 9, Jimi Hendrix

1. Workshops: Last week, your group was assigned a method of approximating \( \pi \). You will have Tuesday’s workshop to work on this project and on Thursday you will present your results. Answer the following questions:

(a) Use your technique to give five increasingly good approximations to \( \pi \).

(b) How far can your method go? If possible, give a formula or an algorithm to find the “nth” approximation.

(c) Is there an error bound for your method? Can you say how accurate each approximation is?

(d) How many steps does your technique take to get two decimal digits of \( \pi \)? Five decimal digits? 100 decimal digits? Would you say it is practical for computing \( \pi \)?

2. Approximate \( \int_0^{\sqrt{\pi}} \sin(x^2) \, dx \) via trapezoids and via Simpson’s rule with \( n = 6 \). Find an error bound in each case. Then find an \( n \) required to make the error smaller than \( 10^{-6} \) in each case. It would be hard to find \( M_2 \) and \( M_4 \) exactly in this problem. Rather than doing this, simply overestimate them.

3. Find an example of an integral \( \int_a^b f(x) \, dx \) where the approximation by trapezoids with \( n = 2 \) is more accurate than Simpson’s rule with \( n = 2 \).


(a) \( \int \frac{1}{\sqrt{x} + \sqrt{x}} \, dx \)

(b) \( \int_0^1 \frac{\arctan(x)}{x+1} \, dx \)

(c) \( \int \frac{1}{1 - \tan^2(x)} \, dx \)

(d) \( \int_0^1 \ln(\sqrt{1+x} + \sqrt{1-x}) \, dx \)

5. Section 13.2, problems 1, 2, 4, 6. For problems 1b, 1f, and 1i, and 1n, given \( \epsilon > 0 \) show how to choose \( n \) so that \( |L - a_n| < \epsilon \).

6. In some cases we say that a sequence “goes to infinity” or that \( a_n \to +\infty \) as \( n \to \infty \). This means more than just that the sequence diverges—for example, the sequence \( (-1)^n \) diverges but does not go to infinity. Give a precise definition of this property.