Instructions Please answer all questions below. Write clearly and circle your final answers. Show your work for partial credit. You can use a calculator, but no notes or books. Good luck.

Useful power series formulas:

\[
\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n
\]
\[
\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}
\]
\[
\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^nx^{2n+1}}{2n+1}
\]
\[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}
\]
\[
\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^nx^{2n+1}}{(2n+1)!}
\]
\[
\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^nx^{2n}}{(2n)!}
\]
\[
(1+x)^p = \sum_{n=0}^{\infty} \frac{p(p-1)\cdots(p-n+1)x^n}{n!}
\]

Miscellaneous results:

Approximation by trapezoids: \(\int_b^a f(x) \, dx \approx \Delta x \left( \frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n \right)\) with error less than \(\frac{M_2(b-a)^3}{12n^2}\), where \(M_2\) is the maximum value of \(|f''(x)|\) in \([a,b]\).

Simpson’s rule: \(\int_a^b f(x) \, dx \approx \Delta x \left( \frac{1}{3}y_0 + \frac{3}{3}y_1 + \frac{3}{3}y_2 + \frac{3}{3}y_3 + \frac{3}{3}y_4 + \cdots + \frac{3}{3}y_{n-2} + \frac{3}{3}y_{n-1} + \frac{1}{3}y_n \right)\) with error less than \(\frac{M_4(b-a)^5}{180n^4}\), where \(M_4\) is the maximum value of \(|f''''(x)|\) in \([a,b]\).

Taylor’s formula and Lagrange’s error bound: \(f(x) \approx f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + 3 + \cdots + \frac{f^{(N)}(0)}{N!}x^N\), with error less than \(\frac{M_{N+1}|x|^{N+1}}{(N+1)!}\), where \(M_{N+1}\) is the maximum value of \(|f^{(N+1)}(x)|\) in \([0,x]\).

Curvature of parametric plane curves: \(k = \frac{x'y''-y'x''}{(x'^2+y'^2)^{3/2}}\).
1. (a) Compute the indefinite integral \( \int \sqrt{1-x^2} \, dx \), and conclude that \( \int_{-1}^{1} \sqrt{1-x^2} \, dx = \frac{\pi}{2} \).

To compute \( \int \sqrt{1-x^2} \, dx \), let \( x = \sin(u) \), then \( dx = \cos(u) \, du \).

\[
\int \sqrt{1-\sin^2 u} \cdot \cos u \, du = \int \cos^2 u \cdot \cos u \, du = \int \cos^3(u) \, du
\]

By double-angle formulas, this is

\[
\int \frac{\cos(2u) + 1}{2} \, du = \frac{\sin(2u)}{4} + \frac{u}{2} + C
\]

\[
= \frac{\sin(u) \cos(u)}{2} + \frac{u}{2} + C
\]

And using \( u = \arcsin(x) \), \( \cos(u) = \sqrt{1-x^2} \), we get

\[
x \frac{\sqrt{1-x^2}}{2} + \arcsin(x) + C
\]

Evaluating at \( u = 1 \), we get

\[
0 + \arcsin(1) = 0 - \arcsin(-1) \quad \frac{\pi}{2} = \frac{\pi}{2} \quad \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}.
\]

(b) Use four trapezoids to approximate this integral. Give the error bound.

We will use four trapezoids as shown:

\[
x_0 = -1, \quad x_1 = -\frac{1}{2}, \quad x_2 = 0, \quad x_3 = \frac{1}{2}, \quad x_4 = 1, \quad \Delta x = \frac{1}{2}
\]

\[
y_0 = 0, \quad y_1 = \frac{\sqrt{3}}{2}, \quad y_2 = 1, \quad y_3 = \frac{\sqrt{3}}{2}, \quad y_4 = 0.
\]

Then the formula for approximation by trapezoids gives us

\[
\frac{1}{2} \left( \frac{1}{2} \cdot 0 + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot 0 \right)
\]

\[
= \frac{1 + \sqrt{3}}{2}.
\]

For the error bound, \( f'(x) = \sqrt{1-x^2}, \) \( f''(x) = \frac{x}{\sqrt{1-x^2}}, \) \( f''(x) = \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{(1-x^2)^{3/2}} \)

Unfortunately, this \( f''(x) \) goes to infinity as \( x \to 1 \), so \( M_2 \) is infinity in this case and the error bound is useless. My mistake.

2. The rose \( r = 2 \sin(2\theta) \) and the circle \( r = \sqrt{3} \) are shown. Find the area inside both the rose and the circle.

The rose and the circle intersect wherever 
\[
2 \sin(2\theta) = \sqrt{3}.
\]
\[
\sin(2\theta) = \frac{\sqrt{3}}{2}
\]
\[
\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}.
\]

Since the rose equation can produce negative values of \( r \), we also must take into account points where 
\[
2 \sin(2\theta) = -\sqrt{3},
\]
\[
\theta = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}.
\]

First I will compute the area between the rose and the circle in the first quadrant (area A).

This is 
\[
\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{2} ((2 \sin(2\theta))^2 - (\sqrt{3})^2) \, d\theta
\]
\[
= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2 \sin^2(2\theta) - \frac{3}{2} \, d\theta
\]
\[
= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 - \cos(4\theta) - \frac{3}{2} \, d\theta = \left[ \frac{\sin(4\theta)}{4} - \frac{\theta}{2} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\sqrt{3}}{4} - \frac{\pi}{12}
\]

Next the area of the rose in the first quadrant (area A+B).

\[
\int_{0}^{\frac{\pi}{12}} \frac{1}{2} (2 \sin(2\theta))^2 \, d\theta = \int_{0}^{\frac{\pi}{12}} 2 \sin^2(2\theta) \, d\theta + \int_{0}^{\frac{\pi}{12}} 1 - \cos(4\theta) \, d\theta
\]
\[
= \theta - \frac{\sin(4\theta)}{4} \bigg|_{0}^{\frac{\pi}{12}} = \frac{\pi}{12} - \frac{\sqrt{3}}{2}
\]

Subtracting these gives area \( B = \frac{7\pi}{12} - \frac{\sqrt{3}}{4} \), and multiplying by 4 gives the final answer, \( \frac{7\pi}{3} - \sqrt{3} \).
3. (a) Show that \( \lim_{n \to \infty} \frac{n}{n^2 - 2} = 0 \) by comparison to a harmonic sequence.

**Claim:** for \( n \geq 2 \), \( 0 < \frac{n}{n^2 - 2} \leq \frac{2}{n} \).

We know that \( n > 0 \) and \( n^2 - 2 > 0 \), so the first inequality is true. We can check the second by cross-multiplying:

\[ n^2 \leq 2n^2 - 4. \]

This is true as long as \( n^2 > 4 \), i.e., for \( n \geq 2 \).

Thus as \( n \to \infty \), \( \frac{n}{n^2 - 2} \) is squeezed between 0 and \( \frac{2}{n} \), both of which approach 0.

So, \( \lim_{n \to \infty} \frac{n}{n^2 - 2} = 0 \).

(b) Show that \( \sum_{n=0}^{\infty} \frac{n}{n^2 - 2} = \infty \) by comparison to a harmonic series.

This time, for \( n \geq 2 \), we use the fact that

\[ \frac{1}{n} < \frac{n}{n^2 - 2} \]

We can test this by cross-multiplying, too:

\[ n^2 - 2 < n^2. \]

Thus, by the comparison test, since

\[ \sum_{n=1}^{\infty} \frac{1}{n} \] diverges, \( \sum_{n=0}^{\infty} \frac{1}{n^2 - 2} \) diverges.
4. The vector function \( \vec{r}(t) = (\cos t + t \sin t, \sin t - t \cos t) \) describes a curve called the involute of a circle. It traces the path of the end of a string as it is unwound from a circular spool. Find the curvature at \( t = \pi \) and \( t = 2\pi \).

\[ x(t) = \cos t + t \sin t \]
\[ x'(t) = -\sin t + \sin t + t \cos t \]
\[ x''(t) = -\cos t - \cos t - t \sin t \]
\[ y(t) = \sin t - t \cos t \]
\[ y'(t) = \cos t - \cos t - t \sin t \]
\[ y''(t) = \sin t + t \cos t \]

By the curvature formula \( k = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}} \)

\[ k(t) = \frac{4 \sin t \cos t + t^2 \cos^2 t + \sin t + t \sin t - t^2 \sin t}{(t^2 \sin^2 t + t^3 \cos^2 t)^{3/2}} \]

\[ = \frac{2 \sin t \cos t}{t^2} + \frac{\cos^2 t - \sin^2 t}{t} \]

At \( t = \pi \), \( k = \frac{1}{\pi} \). At \( t = 2\pi \), \( k = \frac{1}{2\pi} \).
5. Find the radius of convergence for the following power series:

(a) \( \sum_{n=0}^{\infty} \frac{n!}{2(n^2)} x^n \)

Apply the ratio test:

\[
\frac{(n+1)!}{2(n+2)^2} x^{n+1} / \frac{n!}{2n^2} x^n = \frac{n+1}{2n+1} x
\]

As \( n \to \infty \), this approaches \( 0 \), regardless of \( x \).

So this series converges for all \( x \).

(b) \( \sum_{n=0}^{\infty} \frac{n}{3^n} x^{2n+1} \)

\[
\frac{n+1}{3^{n+1}} x^{2n+1} \quad / \quad \frac{n}{3^n} x^{2n+1} = \frac{1}{3} \frac{n+1}{n} x^2
\]

As \( n \to \infty \), this approaches \( \frac{1}{3} x^2 \).

So the series converges for \( |\frac{1}{3} x^2| < 1 \), or \( -\sqrt{3} < x < \sqrt{3} \).

(c) \( \sum_{n=0}^{\infty} \frac{(-1)^n n!}{(n+2)!} x^n \)

\[
\frac{(-1)^{n+1} (n+1)!}{(n+3)!} x^{n+1} / \frac{(-1)^n n!}{(n+2)!} x^n = -\frac{n+1}{n+3} x
\]

As \( n \to \infty \), this approaches \( x \).

So the series converges for \( |x| < 1 \).
6. True or False: (and explain why)

(a) If the series $\sum_{n=0}^{\infty} a_n$ converges, then the sequence $a_n$ converges.

True, in fact $a_n$ must converge to 0, by the $n$th term test.

(b) If the series $\sum_{n=0}^{\infty} a_n$ diverges, then the sequence $a_n$ diverges.

False, for example the sequence $\sum_{n=1}^{\infty} 1$ diverges, but the terms converge to 1.

Also, the series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots$ diverges, but the terms converge to 0.

(c) If the series $\sum_{n=0}^{\infty} a_n 6^n$ converges, then the series $\sum_{n=0}^{\infty} a_n (-2)^n$ converges.

True, since the power series $\sum_{n=0}^{\infty} a_n x^n$ must have radius of convergence at least 6, it also converges at $x = -2$.

(d) If the series $\sum_{n=0}^{\infty} a_n 6^n$ converges, then the series $\sum_{n=0}^{\infty} a_n (-6)^n$ converges.

False, since 6 and -6 could be endpoints of an interval of convergence, and convergence at endpoints is unpredictable. For example, the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n \cdot 6^n} x^n$ converges at $x = 6$ and diverges at $x = -6$. 
7. (a) Write down the terms of the power series of \( f(x) = \sin \left( \frac{3\pi}{4} + x \right) \) up to \( x^3 \).

\[
\begin{align*}
f(0) &= \sin \left( \frac{3\pi}{4} \right) = \frac{\sqrt{2}}{2} \\
\frac{f'(0)}{1!} &= -\cos \left( \frac{3\pi}{4} \right) = -\frac{\sqrt{2}}{2} \\
\frac{f''(0)}{2!} &= -\sin \left( \frac{3\pi}{4} \right) = -\frac{\sqrt{2}}{2} \\
\frac{f'''(0)}{3!} &= \cos \left( \frac{3\pi}{4} \right) = \frac{\sqrt{2}}{2} \\
\end{align*}
\]

\[
\sin \left( \frac{3\pi}{4} + x \right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{x}{1!} - \frac{\sqrt{2}}{2} \cdot \frac{x^2}{2!} + \frac{\sqrt{2}}{2} \cdot \frac{x^3}{3!} + \ldots
\]

(b) How small must \( x \) be to guarantee that these terms approximate \( f(x) \) with error less than \( \frac{1}{1,000,000} \)?

We can use the Lagrange error bound \( \frac{M_k x^k}{k!} \) here.

Since \( M_k \) is a maximum value of \( |f^{(k)}(x)| = |\sin \left( \frac{3\pi}{4} + x \right)| \), we can set \( M_k = 1 \) (i.e., always an upper bound for the \( \sin \) function). Then we want

\[
\frac{1 \cdot \frac{x^4}{4!}}{2} < \frac{1}{1,000,000}
\]

\[
x < \sqrt[4]{\frac{2 \cdot 1}{1000000}}
\]

(c) Suppose we want to approximate \( f(x) \) for \( \frac{2\pi}{3} < x < \frac{5\pi}{6} \) with error less than \( \frac{1}{1,000,000} \). How many terms of the series would we need?

Now we compute the Lagrange error bound \( \frac{M_N x^N}{N!} \) with \( N \) unknown.

Since \( M_N \) is a maximum value for \( |\sin \left( \frac{3\pi}{4} + x \right)| \) or \( |\cos \left( \frac{3\pi}{4} + x \right)| \) with \( \frac{2\pi}{3} < \frac{3\pi}{4} + x < \frac{5\pi}{6} \), \( M_N \) will always be \( \frac{\sqrt{2}}{2} \). (We could also take \( M_N = 1 \), just to be safe, but \( \frac{\sqrt{2}}{2} \) gives a more precise answer).

Here \( x \) is at most \( \frac{\pi}{12} \). So we want

\[
\frac{\frac{\sqrt{2}}{2} \cdot \left( \frac{\pi}{12} \right)^{N+1}}{(N+1)!} < 1000000.
\]

This occurs at \( N = 5 \), so 5 terms are required.
8. The lemniscate \( r^2 = 2\cos(2\theta) \) is shown. Find the tangential angle \( \psi \) when \( \theta = \pi \) and verify that the tangent line is vertical. Find the angle \( \psi \) when \( \theta = \frac{\pi}{6} \) and compute the slope of the tangent line there.

Recall that \( \tan \psi = \frac{1}{r} \frac{dr}{d\theta} \).

At \( \theta = \pi \), by implicit differentiation,

\[
2r \frac{dr}{d\theta} = -4 \sin(2\theta) = 0
\]

So \( \frac{dr}{d\theta} = 0 \), and \( \tan \psi = 0 \),

so \( \psi = 0 \).

Thus the tangent line is orthogonal to the \(-x\) axis, so it is vertical.

At \( \theta = \frac{\pi}{6} \), \( r = 1 \). So \( 2 \cdot 1 \cdot \frac{dr}{d\theta} = -4 \sin \left( 2 \cdot \frac{\pi}{6} \right) \). Thus \( \frac{dr}{d\theta} = -\sqrt{3} \).

\[
\tan \psi = \frac{1}{r} \frac{dr}{d\theta} = -\sqrt{3}, \quad \text{so} \quad \psi = -\frac{\pi}{3}.
\]

Here is an illustration of the tangent line:

The angle between the tangent line and the \( x\)-axis would be \( \frac{\pi}{6} + \frac{\pi}{2} - \left( -\frac{\pi}{3} \right) = \pi \),

so the tangent line is parallel to the \( x\)-axis.

Its slope is \( 0 \).
9. A point travels along a parabolic path with position function \( \vec{r}(t) = \hat{i}t + t^2\hat{j} \).

(a) Find the tangential and normal components of its acceleration at the point \( 3\hat{i} + 9\hat{j} \).

\[
\vec{r}(0) = \hat{i} + 0\hat{j}, \quad \vec{r}(4) = 3\hat{i} + 9\hat{j}; \quad \vec{r}'(4) = \hat{i} + 2\hat{j}; \quad \vec{r}''(4) = 2\hat{j}.
\]

Speed \( \frac{ds}{dt} = \sqrt{\vec{r}'(4)} = \sqrt{1 + 4^2} = \sqrt{17} \)

Rate of change of speed \( \frac{d^2s}{dt^2} = \frac{4}{\sqrt{1 + 4^2}} \)

Unit tangent vector \( \hat{T} = \frac{\vec{r}'(4)}{\sqrt{\vec{r}'(4)}} = \frac{\hat{i} + 2\hat{j}}{\sqrt{1 + 4^2}} \)

The tangential component of acceleration is \( \frac{d^2s}{dt^2} \hat{T} = \frac{12}{\sqrt{17}} \left( \frac{\hat{i}}{\sqrt{17}} + \frac{6\hat{j}}{\sqrt{17}} \right) \)

The normal component is \( 2\hat{j} \left( \frac{12}{37} + \frac{72}{37} \right) = \frac{12\hat{i}}{37} + \frac{72\hat{j}}{37} \)

(b) What if instead it were moving at a constant speed of 2 along the curve? In that case, find the tangential and normal components of its acceleration at \( 3\hat{i} + 9\hat{j} \).

Since the speed is constant, the tangential component of acceleration would be 0.

The normal component is curvature \( \kappa \), so we need to compute the curvature.

\[
k = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}} = \frac{1 \cdot 2 - 2 \cdot 0}{(1 + 4^2)^{3/2}} = \frac{2}{(1 + 4)^{3/2}}
\]

at \( t = 3 \), \( k = \frac{2}{37^{3/2}} \).

Thus the normal component has magnitude \( \frac{2}{37^{3/2}} \cdot 2 = \frac{8}{37^{3/2}} \).

The unit normal vector is \( \hat{n} \) rotated counterclockwise by \( \frac{\pi}{2} \), giving \( \frac{-6\hat{i}}{\sqrt{37}} + \frac{3\hat{j}}{\sqrt{37}} \).

So the normal component of acceleration is \( \frac{8}{37^{3/2}} \left( \frac{-6\hat{i}}{\sqrt{37}} + \frac{3\hat{j}}{\sqrt{37}} \right) = \frac{-48\hat{i}}{37^{3/2}} + \frac{8\hat{j}}{37^{3/2}} \).
10. Explain why the following integrals are improper, and compute them step-by-step:

(a) \[ \int_{0}^{3} \ln(x)^2 \, dx \quad \text{Improper b/c } \ln(x) \to -\infty \text{ as } x \to 0. \]

\[
\lim_{a \to 0^+} \int_{a}^{3} \ln(x)^2 \, dx \quad \text{Integrate by parts with } f'(x) = \ln(x), \quad g(x) = \frac{x^2}{2} \\
= \lim_{a \to 0^+} \left[ x \ln(x)^2 \right]_{a}^{3} - \int_{a}^{3} 2x \ln(x) \, dx = \lim_{a \to 0^+} \left[ x \ln(x)^2 - 2x \ln(x) + 2x \right]_{a}^{3} \\
= 3 \cdot 3^2 - 2 \cdot 3 \ln(3) + 2 \cdot 3 - \left( a \ln(a)^2 - 2a \ln(a) + 2a \right) \\
= 27 - 6 \ln(3) + 6 - a \ln(a)^2 + 2a \ln(a) - 2a. 
\]

Using L'Hôpital, we can show that \( \lim_{a \to 0} a \ln(a)^2 = \lim_{a \to 0} a \ln(a) = 0. \)

So the final answer is \( 27 - 6 \ln(3) + 6 + 0 - 0 = 33 \).

(b) \[ \int_{0}^{\infty} x^2 e^{-x} \, dx \quad \text{Improper b/c upper endpoint is } \infty. \]

\[
\lim_{b \to \infty} \int_{0}^{b} x^2 e^{-x} \, dx \quad \text{Int. by parts with } f'(x) = x^2, \quad g(x) = -e^{-x} \\
= \lim_{b \to \infty} \left[ x^2 e^{-x} \right]_{0}^{b} + \int_{0}^{b} 2x e^{-x} \, dx, \quad \text{Int. by parts again with } f'(x) = 2x, \quad g(x) = -e^{-x} \\
= \lim_{b \to \infty} -x^2 e^{-x} \Big|_{0}^{b} - 2x e^{-x} \Big|_{0}^{b} + \int_{0}^{b} 2e^{-x} \, dx \\
= \lim_{b \to \infty} -x^2 e^{-x} \Big|_{0}^{b} - 2x e^{-x} \Big|_{0}^{b} - 2e^{-x} \Big|_{0}^{b} \quad \text{By L'Hôpital, this is simply} \\
= 0 + 0 + 0 + 0 + 0 + 0 + 2e^0 = 2. 
\]

(c) \[ \int_{1}^{\infty} \frac{1}{x^2 + 2x} \, dx \quad \text{Improper b/c upper endpoint is } \infty. \]

Do partial fractions: \( \frac{1}{x(x+2)} = \frac{a}{x} + \frac{b}{x+2} \Rightarrow a + b = 0 \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}, \ b = -\frac{1}{2}. \)

\[
\lim_{b \to \infty} \int_{1}^{b} \left( \frac{1/2}{x} - \frac{1/2}{x+2} \right) \, dx = \lim_{b \to \infty} \left[ \frac{1}{2} \ln(x) \right]_{1}^{b} - \frac{1}{2} \ln(x+2) \right|_{1}^{b} \\
= \lim_{b \to \infty} \left[ \frac{1}{2} \ln \left( \frac{x}{x+2} \right) \right]_{1}^{b} = \frac{1}{2} \ln(1) - \frac{1}{2} \ln \left( \frac{1}{3} \right) \\
= -\frac{1}{2} \ln \left( \frac{1}{3} \right). 
\]
11. A point on the wheel of a car moves with position function \( \vec{r}(t) = (50t - \sin(50t), 1 - \cos(50t)) \).

(a) Find its velocity and acceleration at time \( t \)

\[
\vec{v}(t) = \left( 50 - 50 \cos(50t), \quad 50 \sin(50t) \right)
\]

\[
\vec{a}(t) = \left( 2500 \sin(50t), \quad 2500 \cos(50t) \right)
\]

(b) Find its speed and the rate of change of its speed at time \( t \).

\[
\text{speed} \quad \left| \vec{v}(t) \right| = \sqrt{(50 - 50 \cos(50t))^2 + (50 \sin(50t))^2}
\]

\[
= 50 \sqrt{1 - 2 \cos(50t) + \cos^2(50t) + \sin^2(50t)}
\]

\[
= 50 \sqrt{2 - 2 \cos(50t)}
\]

\[
\frac{\text{d}}{\text{dt}} \text{ of this} = \frac{2500 \sin(50t)}{\sqrt{2 - 2 \cos(50t)}}
\]
12. Using the fact that \( \arccos(x) = -\int_x^1 (1 - t^2)^{-1/2} \, dt \), derive the first five nonzero terms of the Taylor series of \( \arccos(x) \) from a binomial series.

The binomial series for power \(-\frac{1}{2}\) is

\[
(1 + x)^{-\frac{1}{2}} = 1 + \frac{-\frac{1}{2}}{1!} x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} x^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!} x^3 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})}{4!} x^4 + \ldots
\]

Replacing \( x \) with \(-x^2\) and simplifying:

\[
1 + \frac{1}{2} x^2 + \frac{3}{8} x^4 + \frac{5}{16} x^6 + \frac{35}{128} x^8 + \ldots
\]

This is the power series for \((1 + x^2)^{-1/2}\).

If we integrate it term by term,

\[
\int_0^1 \left( 1 + \frac{1}{2} t^2 + \frac{3}{8} t^4 + \frac{5}{16} t^6 + \frac{35}{128} t^8 + \ldots \right) \, dt
\]

\[
= 1 + \frac{1}{6} t^3 + \frac{3}{40} t^5 + \frac{5}{112} t^7 + \frac{35}{1152} t^9 + \ldots \bigg|_0^1
\]

\[
= 1 - \frac{1}{6} x^3 - \frac{3}{40} x^5 - \frac{5}{112} x^7 - \ldots
\]

The value of \( C \) may seem mysterious, but since this is the Taylor series of \( \arccos \), \( C \) must be \( \arccos(0) = \pi \).

So we have

\[
1 - \frac{1}{6} x^3 - \frac{3}{40} x^5 - \frac{5}{112} x^7 - \ldots
\]