Minnesota themed trivia & a Hill cipher

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This puzzle will test your knowledge of fun facts about The U and the state of Minnesota as a whole! Please follow the directions below, carefully solving each of the trivia questions. You will want to use an internet search engine to find specific names, dates, etc. in order to ensure the accuracy of your solutions. Then, you will arrange these integers in a 3x3 matrix (Hint: this is the key) that has been used to encrypt some plaintext. You are given the cipher text and using your key from the previous step, you must solve for the original plaintext.

Directions
Each of the nine trivia questions below yields a unique integer mod 26. Please solve for the integers and arrange them in a 3x3 encryption matrix equivalent to the following:

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\]

(1) in the matrix above refers to the answer resulting from deciphering trivia question 1 (please follow this pattern for inputting answers 2-9 into the matrix). The result will be a 3x3 encryption matrix that will be used to decipher the following:

K Y O K U R C F A

*please keep in mind, the matrix whose entries are the integers resulting from solving the trivia questions is your encryption matrix and you must invert it mod 26 in order to decipher the text above.

Trivia questions
(1) What renowned architect designed the Weisman Art Museum and when was the Weisman Art Museum established? Take the sum of the architect's name and find that number's inverse modulo the year the Weisman was established. Finally, take that number modulo 26.
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Solution

For our cryptology puzzle, each group member created three unique questions to be deciphered using an algorithm or theorem introduced in the first six weeks of class. Solving each question resulted in a unique integer and we selected nine of those integers to be entered into $K$, an invertible $3 \times 3$ encryption matrix. Since it's Homecoming Weekend, we chose "Go Gophers" for our plaintext and arranged the letters in a $3 \times 3$ matrix to be encrypted with $K$. Multiplying our plaintext and encryption matrix yielded the cipher text: "K Y O K U R C F A".

Below you will find the solution for our nine trivia questions as well as the solution for backing into our plaintext. We inverted our key, $K$, mod 26 using the signed, transposed matrix of minors and the equation: $A^{-1} = B(\text{det} A)^{-1}$. Multiplying the inverse of our key by the ciphertext arranged in a $3 \times 3$ matrix yields the plaintext: "Go Gophers".

Trivia questions
(1) Frank Gehry, 1934

\[
\begin{align*}
\text{frank gehry} &= 5 + 17 + 0 + 13 + 10 + 6 + 4 + 7 + 17 + 24 = 103 \\
\text{Find: } 103x & \equiv 1 \text{ mod } 1934 \\
1934 &= 18(103) + 80 \\
103 &= 1(80) + 23 \\
80 &= 3(23) + 11 \\
23 &= 2(11) + 1 \\
11 &= 1(11) + 0 \\
80 &= 1934 - 18(103) \\
23 &= 103 - 80 = 103 - (1934 - 18(103)) = 19(103) - 1934 \\
11 &= 80 - 3(23) = 1934 - 18(103) - 3(19(103) - 1934) = 4(1934) - 75(103) \\
1 &= 23 - 2(11) = 19(103) - 1934 - 2(4(1934) - 75(103)) = 169(103) - 9(1934) \\
169 &\equiv 13 \text{ mod } 26
\end{align*}
\]

(2) (Rochester) = (17, 14, 2, 7, 4, 18, 19, 4, 17)
Sum = 102 ≡ 24 mod 26

Answer is 24

(3) Truck name: "Tonka" = 19 + 14 + 13 + 10 + 0 = 56
    Town name: "Minnetonka" = 12 + 8 + 13 + 13 + 4 + 19 + 14 + 13 + 10 + 0 = 106
    \( GCD(56,106) = 2 \)

(4) "Holy Cow" = 7 + 14 + 11 + 24 + 2 + 14 + 22 = 94
    \[ 2^{94} \equiv 18 \text{ mod } 23 \]

(5) 1851, 2016
    2016 = 1(1851) + 165
    1851 = 11(165) + 36
    165 = 4(36) + 21
    36 = 1(21) + 15
    21 = 1(15) + 6
    15 = 2(6) + 3
    6 = 2(3) + 0

    \( GCD(1851,2016) = 3 \)

(6) State muffin is "Blueberry"
    (Blueberry) = (1, 11, 20, 4, 1, 4, 17, 17, 24)
    Sum = 99

    \( 7^{99} \text{ mod } 26 \) via fast exponentiation:
    \[ 7^1 = 7 \ ; \ 7^2 = 23 \ ; \ 7^4 = 9 \ ; \ 7^8 = 3 \ ; \ 7^{16} = 9 \ ; \ 7^{32} = 3 \ ; \ 7^{64} = 9 \]
    99 = 64 + 32 + 2 + 1

    \[ 7^{99} = (7^{64})(7^{32})(7^2)(7^1) = (9)(3)(23)(7) = 4347 \equiv 5 \text{ mod } 26 \]

(7) Eddy Hall, 1886
    eddy hall = 4 + 3 + 3 + 24 + 7 + 0 + 11 + 11 = 63
    \[ 63 \text{ mod } 26 = 11 \]
    \[ 11^{1886} \text{ mod } 26 \]
    \[ Mod 26: \]
$11^1 \equiv 11$
$11^2 \equiv 17$
$11^4 \equiv 17^2 \equiv 3$
$11^8 \equiv 3^2 \equiv 9$
$11^{16} \equiv 9^2 \equiv 3$
$11^{32} \equiv 3^2 \equiv 9$
$11^{64} \equiv 9^2 \equiv 3$
$11^{128} \equiv 3^2 \equiv 9$
$11^{256} \equiv 9^2 \equiv 3$
$11^{512} \equiv 3^2 \equiv 9$
$11^{1024} \equiv 9^2 \equiv 3$

$1886 = 1024 + 512 + 256 + 64 + 16 + 8 + 4 + 2$
$111886 = 111024x + 11512x + 11256x + 1164x + 1116118x + 114x + 112$

$= 3 \times 9 \times 3 \times 9 \times 3 \times 9 \times 3 \times 17 = 334611 \equiv 17 \mod 26$

(8) First Hockey Championship: 1974
Last Football Championship: 1960

\[ \text{GCD}(1974, 1960): \]
\[ 1974 = 1(1960) + 14 \]
\[ 1960 = 140(14) + 0 \]
Answer is 14

(9) The Twin Cities metro is home to 17 Fortune 500 companies

\[ 17500 \mod 26 \]
\[ 26 = 8 \times 3 + 2 \]
\[ 3 = 1 \times 2 + 1 \]
\[ 2 = 2 \times 1 + 0 \]
\[ (3 \times 9) + (-1 \times 26) = 1 \]
Therefore, $3^{-1} \equiv 9 \mod 26$
Answer is 9
Cipher text decryption

The encryption matrix is:

\[ K = \begin{pmatrix} 13 & 24 & 2 \\ 18 & 3 & 5 \\ 17 & 14 & 9 \end{pmatrix} \]

In order to decrypt the cipher text groups will need to invert this matrix mod 26. The determinant of \( K \) is -2005 which is congruent to 23 mod 26. The multiplicative inverse of 23 mod 26 is 17.

The decryption is as follows:

\[ \text{matrix of minors:} \quad \begin{pmatrix} -43 & 77 & 201 \\ 188 & 83 & -226 \\ 114 & 29 & -393 \end{pmatrix} \]

which is congruent to:

\[ \begin{pmatrix} 9 & 25 & 19 \\ 6 & 5 & 8 \\ 10 & 3 & 23 \end{pmatrix} \mod 26 \]

signed matrix of minors:

\[ \begin{pmatrix} 9 & -25 & 19 \\ -6 & 5 & -8 \\ 10 & -3 & 23 \end{pmatrix} \]

transposed:

\[ \begin{pmatrix} 9 & -6 & 10 \\ -25 & 5 & -3 \\ 19 & -8 & 23 \end{pmatrix} \]

which is congruent to:

\[ \begin{pmatrix} 9 & 20 & 10 \\ 1 & 5 & 23 \\ 19 & 28 & 23 \end{pmatrix} \]

since \( 23^{-1} \equiv 17 \mod 26 \), multiply: \( 17 \times \begin{pmatrix} 153 & 340 & 170 \\ 17 & 85 & 391 \\ 323 & 306 & 391 \end{pmatrix} \) and reduce mod 26, this yields the following decryption matrix:

\[ \begin{pmatrix} 23 & 2 & 14 \\ 17 & 7 & 1 \\ 11 & 20 & 1 \end{pmatrix} \]

The cipher text is:

\[ \text{K Y O K U R C F A} \]
\[ \text{10 24 14 10 20 17 2 5 0} \]

which can be written in a 3x3 matrix:

\[ A = \begin{pmatrix} 10 & 10 & 2 \\ 24 & 20 & 5 \\ 14 & 17 & 0 \end{pmatrix} \]
Multiply the decryption matrix and cipher text matrix:

\[
\begin{pmatrix}
474 & 508 & 56 \\
352 & 327 & 69 \\
604 & 527 & 122
\end{pmatrix}
\]

and reduce mod 26:

\[
\begin{pmatrix}
6 & 14 & 4 \\
14 & 15 & 17 \\
6 & 7 & 18
\end{pmatrix}
\]

Which spells out:

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 g o g o p h e r s
 6 14 6 14 15 7 4 17 18
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