18.089 Homework 2

Summer 2010

Due Monday, June 14

1. Graph the curve defined by the parametric equations \( y = \sin t \), \( x = \sin t \) as \( t \) varies over all of \( \mathbb{R} \). How would you describe this curve in rectangular coordinates (i.e., without using a parameter)?

2. At time \( t \), a particle has position vector given by \( \mathbf{R}(t) = (\cos t, \sin t, t) \). Compute its velocity, acceleration and speed as functions of \( t \).

3. Let \( \ell \) be the line passing through the points \((0, 1)\) and \((1, 0)\) (in rectangular coordinates). Given three equations for this line: one in rectangular coordinates, one in parametric coordinates and one in polar coordinates.

4. What is the intersection of the line \( \frac{x-1}{1} = \frac{y}{2} = \frac{z+2}{2} \) and the plane \( x + y + z = 3 \)?

5. Compute the arclength of a single arch of the cycloid generated by a circle of radius \( r \).

6. Suppose a particle moves according to \( \mathbf{R}(t) = e^t \cos t \, \hat{i} + e^t \sin t \, \hat{j} \). Compute its velocity and acceleration vectors. Then find its speed, and the magnitude of the tangential and normal components of acceleration.

7. The curve given in polar coordinates by \( r = 1 - \sin \theta \) is called a \textit{cardioid}.
   
   (a) Sketch the cardioid.
   
   (b) Compute its arclength.
   
   (c) Compute the area that it bounds.

   (You may want to use the half-angle formula \( \sqrt{\frac{1-\cos \theta}{2}} = \sin \frac{\theta}{2} \) for \( 0 \leq \theta \leq 2\pi \). (For other values of \( \theta \), the two sides may disagree by a sign.))

8. The curve given in polar coordinates by \( r = 1 + 2 \sin \theta \) is called a \textit{limaçon}. Sketch it – note in particular the existence of an inner and outer loop. Compute the area bounded by the inner loop.

9.