

Math 5385 - Spring 2019
Problem Set 1

Submit solutions to **four** of the following problems.

1. Complete the first two sections of the Macaulay 2 tutorial at <http://web.macaulay2.com/> (Welcome Tutorial and Basic Introduction), Answer the following questions, including code.
 - (a) Use M2 to factor the number 738,399,887
 - (b) Use M2 to list all perfect numbers (sum of proper positive divisors equals n) less than 100
2. This exercise from the textbook: [IVA] §1.1 #2 (p. 5)
3. Use MathSciNet (through a UMN proxy), the arXiv, and MathOverflow to answer the following questions:
 - (a) Estimate the number of journal articles published with the words “Gröbner basis” in their title.
 - (b) How many algebraic geometry preprints were added to the e-print archives in November 2018?
 - (c) Estimate the number of research level math questions tagged with `ag.algebraic-geometry`.
4. (a) Show that the polynomial

$$\binom{x}{d} := \frac{x(x-1)\cdots(x-d+1)}{d!}$$

takes integer values for all integers x .

- (b) Show that every polynomial of degree d which takes integer values for all integers can be written as a unique integer linear combination of $\binom{x}{d}, \binom{x}{d-1}, \dots, \binom{x}{0}$.
5. Given a sequence $a_n, a_n - 1, \dots, a_m$ of real numbers with $n > m$ and $a_n \neq 0$, the *number of sign changes* is defined as follows: count one sign change if $a_i a_k < 0$ with $k = i - 1$ or $k < i - 1$ and $a_j = 0$ for every j satisfying $k < j < i$.
 - (a) Prove Descartes’s rule of signs: If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_m x^m \in \mathbb{R}[x]$ with $a_n a_m \neq 0$, then the number of positive roots of f is less than or equal to the number of sign changes in the sequence of coefficients $a_n, a_n - 1, \dots, a_m$.
Hint. Proceed by induction, consider the derivative, and use Rolle’s Theorem.
 - (b) Verify that $x^{11} + x^8 - 3x^5 + x^4 + x^3 - 2x^2 + x - 2$ has at most 5 positive and 2 negative roots. Deduce that it has at least 4 nonreal roots.