Math 5385 - Spring 2019
Problem Set 10

Submit solutions to **three** of the following problems.

1. Consider the ideal $I := \langle x^2, xy \rangle$ in $\mathbb{C}[x,y]$. For any $c \in \mathbb{C}$, prove that $I = \langle x \rangle \cap \langle x^2, y - cx \rangle$ is an irredundant primary decomposition of I.

2. Let I be a monomial ideal in $S := \Bbbk[x_1, \ldots, x_n]$.

   (a) Suppose that x^u is a minimal generator of I such that $x^u = x^{v_1} x^{v_2}$, where the monomials x^{v_1} and x^{v_2} are relatively prime. Show that

   $$I = (I + \langle x^{v_1} \rangle) \cap (I + \langle x^{v_2} \rangle) \,.$$

   (b) Find an irredundant primary decomposition of $\langle x^3y, x^3z, xy^3, y^3z, xz^3, yz^3 \rangle$.

3. Let $V = V(y - x^2, z - x^3)$ (the twisted cubic) and let $\phi_1, \phi_2 : V \to \mathbb{R}^2$ be polynomial maps given by $\phi_1(x,y,z) = (2x^2 + y^2, z^2 - y^3 + 3xz)$ and $\phi_2(x,y,z) = (2y + xz, 3y^2)$. Show that ϕ_1 and ϕ_2 are the same polynomial map on the twisted cubic.

4. Problem 5.2.1 from the textbook [IVA]