**Math 5385 - Spring 2019**
**Problem Set 12**

Submit solutions to **four** of the following problems.

1. 5.1.11(a,b,c) from [IVA]
2. 5.2.15 from [IVA](This is the example I did from class in more detail)
3. Show by induction that $\mathbb{P}^n(\mathbb{Z}/p\mathbb{Z})$ has exactly $\sum_{i=0}^{n} p^i$ points. (**Hint.** Show that $\mathbb{P}^n \setminus U_0 \cong \mathbb{P}^{n-1}$)
4. Let $I \subset \mathbf{k}[x_0, \dots x_n]$ be a homogenous ideal. Let $G$ be a homogeneous set of generators for $I$. Show that $G$ is a Groebner basis for $I$ with respect to the lex order if and only if it is a Groebner basis for $I$ with respect to grlex (Note: note grevlex)
5. Let $I_1, \dots, I_s \subset \mathbf{k}[x_0, \dots, x_n]$ be homgenous ideals, and let $V_i = V(I_i) \subset \mathbb{P}^n$ be the corresponding projective varieties.
   (a) Show that $V(I_1 + \cdots + I_s) = \bigcap_{i=1}^{s} V_i$
   (b) Show that $V(I_1 \cap \cdots \cap I_s) = V(I_1 \cdots I_s) = \bigcup_{i=1}^{s} V_i$