Math 5385 - Spring 2019
Problem Set 9

Submit solutions to **three** of the following problems.

1. Find the Zariski closure of the following sets:
   (a) The projection of the hyperbola $V(xy-1)$ in $\mathbb{R}^2$ onto the x-axis.
   (b) The boundary of the first quadrant in $\mathbb{R}^2$.
   (c) The set $\{(x, e^x) | x \in \mathbb{R}\}$
2. Let $I, J, K \subseteq \Bbbk[x_1, \ldots, x_n]$ be ideals. Prove the following:
   (a) $IJ \subseteq K$ if and only if $I \subseteq K : J$.
   (b) $(I : J) : K = I : JK$.
3. Let $I, J \subseteq \Bbbk[x_1, \ldots, x_n]$ be ideals.
   (a) Prove that $I : J^\infty = I : J^N$ if and only if $I : J^N = I : J^{N+1}$. Then use this to describe an algorithm for computing the saturation $I : J^\infty$ based on the algorithm for computing ideal quotients.
   (b) Show that N can be arbitrarily large in $I : J^\infty = I : J^N$. **Hint.** Try $I = \langle x^N(y-1) \rangle$.
4. (a) Show that the intersection of any collection of prime ideals is radical.
   (b) Show that an irredundant intersection of at least two prime ideals is never prime.