# MATH 5385 SPRING 2019 PROJECT

## 1. Structure

- Each student is expected to do a project on a different topic/theorem.
- Each student will give a short (10min) presentation in class with the goal of explaning the theorem, its proof and its relevance. This presention should be targeted at your fellow students and include examples.
- Each student will submit a final report/paper. This should be a self contained document explaining the theorem, again the target audience is your classmates. The paper should describe state and proof the theorem, and give some examples explaining how the theorem works and how it is relevant.
- The paper must be typed, at most 8 pages, 1in margins, reasonable font. I strongly encourage you to use LaTeX.
- You're encouraged to use M2 to create examples, and explore the behavior of the topic you're working on.

## 2. Deadlines

Presentation days will be determined later in the semester, but will be the last few days of classes.

| Date | Item | Weight |
|---|---|---|
| Feb 11 | topic choice (by email) | 2% |
| Feb 25 | outline | 8% |
| March 25 | draft | 10% |
| April 1 | feedback | 10% |
| April 24-May 6 | presentations | 30% |
| April 24-May 6 | presentation feedback | 10% |
| May 6 | Final paper | 30% |

## 3. Project Suggestions

Roughly speaking, you could choose to study about 10-15 pages in any of the books in the references. Here are some specific ideas. It is important to note that these topics generally contain far more material than would be appropriate for your project. That means that your job is to identify the results that you think are most interesting and then present them in an accessible and condensed manner.

If you're not sure what to do, try looking up some of the theorems and having a look. Also, feel free to come to office hours and ask about topics, or email me any questions.

If you have something not on this list you'd like to do please contact me before the topic deadline, this list is not intended as a limitation, but I do want to make sure that you choose something both interesting and reasonable.

- algebraic statistics: [St2] §5 (Theorem 5.3); [St3] §8 (Theorem 8.14)
- Alexander duality: [MS] §5 (Theorem 5.24); [HH] §8.1 (Theorem 8.1.6)

- automatic theorem proving: [IVA] §6.4 (Proposition 6.5.8)
- Barvinok's theorem: [MS] §12.4 (Theorem 12.18)
- Bernstein's theorem: [UAG] §7.5 (Theorem 7.5.4); [St3] §3 (Theorem 3.2)
- Brion's formula: [MS] §12.3 (Theorem 12.13)
- combinatorial Nullstellensatz: [TV] §9.1 (Theorem 9.2)
- computation in local rings: [UAG] §4 (Theorem 4.2.2, Theorem 4.4.2); [GP] §6 (Theorem 6.2.6)
- Fröberg's theorem: [HH] §9.2 (Theorem 9.2.3)
- generic initial ideals: [E] §15.9 (Theorem 15.18); [HH] §4 (Theorem 4.1.2)
- Grassmannians: [H] §11 (Proposition 11.30)
- Hilbert schemes: [MS] §18.2 (Theorem 18.7)
- Hilbert syzygy theorem: [UAG] §6 (Theorem 6.2.1) [E] §15.5 (Theorem 15.10)
- integer programming: [UAG] §8.1-2 (Theorem 8.1.11); [St2] §5 (Theorem 5.5)
- invariant theory of finite groups: [IVA] §7 (Theorem 7.3.5); [St1] §2 (Theorem 2.1.3)
- Ishida complex: [MS] §13.3 (Theorem 13.24)
- Koszul complex: [E] §17.1 (Theorem 17.1); [GP] §7 (Theorem 7.6.14)
- linear partial differential equations: [St3] §10 (Theorem 10.3)
- multigraded Hilbert series: [MS] §8.3 (Theorem 8.20)
- Noether normalization: [E] §13.1 (Theorem 13.3)
- Puiseux series: [E] §13.3 (Corollary 13.15); [St3] §1.4 (Theorem 1.7)
- Quillen–Suslin theorem: [UAG] §5.1 (Theorem 5.1.8); [BG] §8 (Theorem 8.5)
- resolutions of monomial ideals: [MS] §3.5 (Theorem 3.17); [HH] §7 (Theorem 7.1.1)
- resultants: [UAG] §3 (Theorem 3.2.3); [St3] §4 (Theorem 4.6)
- SAGBI basis: [St2] §11 (Theorem 11.4); [MS] §14.3 (Theorem 14.11)
- saturations of affine semigroup rings: [MS] §7.3 (Proposition 7.25)
- Stickelberger's theorem: [UAG] §2 (Theorem 2.4.5); [St3] §2.3 (Theorem 2.6)
- straightening laws: [St1] §3 (Theorem 3.1.7, Theorem 3.2.1)
- sums of squares: [St3] §7 (Theorem 7.3)
- tropical hypersurfaces: [St3] §9 (Theorem 9.17)
- triangulations and toric ideals: [St2] §8 (Theorem 8.3); [BG] §7 (Theorem 7.18)
- universal Gröbner bases: [St2] §1 (Theorem 1.4), §7 (Theorem 7.1)

## References

[BG] Winfried Bruns and Joseph Gubeladze, *Polytopes, rings, and K-theory, Monographs in Mathematics*. Springer, 2009, ISBN 978-0-387-76355-2.

[IVA] David A. Cox, John B. Little, and Don O'Shea, *Ideals, Varieties, and Algorithms*, third edition, Springer, 2007, ISBN 978-0-387-35650-1.

[UAG] David A. Cox, John B. Little, and Don O'Shea, *Using Algebraic Geometry*, GTM 185, Springer, 2005, ISBN 978-0-387-20733-9.

[E] David Eisenbud, *Commutative algebra with a view towards algebraic geometry*, GTM 150. Springer, 1995, ISBN 0-387-94268-8.

[GP] Gert-Martin Greuel and Gerhard Pfister, *A Singular introduction to commutative algebra*, 2nd edition, Springer, 2008, ISBN 978-3-540-73541-0.

[H] Brendan Hassett, *Introduction to Algebraic Geometry*, Cambridge University Press, 2007, ISBN 978-0-521-69141-3.

[HH] Jürgen Herzog and Takayuki Hibi, *Monomial ideals*, GTM 260. Springer, 2011, ISBN 978-0-85729-105-9.

[MS] Ezra Miller and Bernd Sturmfels, *Combinatorial commutative algebra*, GTM 227. Springer, 2005, ISBN 0-387-22356-8.

[St1] Bernd Sturmfels, *Algorithms in invariant theory*, Texts and Monographs in Symbolic Computation. Springer, 1993, ISBN 3-211-82445-6.

[St2] Bernd Sturmfels, *Gröbner bases and convex polytopes*, University Lecture Series 8. American Mathematical Society, 1996, ISBN 0-8218-0487-1.

[St3] Bernd Sturmfels, *Solving systems of polynomial equations*, CBMS 97, American Mathematical Society, 2002, ISBN 0-8218-3251-4.

[TV] Terence Tao and Van H. Vu, *Additive combinatorics*, Cambridge University Press, 2010, ISBN 978-0-521-13656-3.