Math 126 Midterm Name:____________________

1. Determine whether the following statements are true or false. No justification is required. [12 points]

   (a) A PDE of the form $\mathcal{L}(u) = F(u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0$ is linear provided

   $$\mathcal{L}(au + v) = a\mathcal{L}(u) + \mathcal{L}(v)$$

   for all real numbers a and functions $u(x, y)$ and $v(x, y)$.

   (b) The Fourier series for a function f converges uniformly provided $\int_{-\pi}^{\pi} f(x)^2\,dx < \infty$.

   (c) If $u(x, t)$ is a solution of the heat equation on the rectangle $0 \le x \le 1$ and $0 \le t \le 1$, then the maximum of u over the rectangle must be attained on the base $0 \le x \le 1$ and $t = 0$ of the rectangle.

   (d) Let $u(x, t)$ be the solution of the wave equation on the entire real line $-\infty < x < \infty$ with initial position $u(x, 0) = \varphi(x)$ and initial velocity $u_t(x, 0) \equiv 0$. If $\varphi(x) = 0$ for all $|x| \ge 1$, then $\lim_{t\to\infty} u(x, t) = 0$ for every x (i.e., $u \to 0$ pointwise as $t \to \infty$).

2. Find the solution of the linear PDE $yu_x - xu_y = 0$ on $\mathbb{R}^2$ that satisfies $u(x, x) = x^4$ for all $x \geq 0$. [8 points]

3. Solve the wave equation $u_{tt} - u_{xx} = 0$ on the entire real line $-\infty < x < \infty$ with initial position $u(x, 0) \equiv 0$ and initial velocity $u_t(x, 0) = \frac{4x}{x^2+1}$. Simplify your expression for u as much as possible. [10 points]

4. Solve the heat equation $u_t - u_{xx} = 0$ on the infinite strip $0 < x < \pi$ and $0 < t < \infty$ with homogeneous Neumann boundary conditions $u_x(0,t) = u_x(\pi,t) = 0$ for $t > 0$ and initial condition $u(x,0) = x + \cos(2x)$. It may be useful to recall that

$$\int_0^\pi \cos^2(nx)\,dx = \frac{\pi}{2} \qquad (n \in \mathbb{N})$$

[10 points]