Math 126 Midterm Information

- The midterm will take place on Friday, October 9, during class.
- The exam will cover everything up to and including the lecture on Friday Sept 25.
- The exam is closed book. No textbooks, notes, or calculators are allowed.
- The exam will have 4 questions. The first 2 will be short, and the last 2 will be longer and slightly more involved. Below are a collection of sample midterm questions for you to practice.

Sample questions

1. For each of the following equations state whether it is nonlinear, linear homogeneous, or linear inhomogeneous.

   (a) $u_{ttt} + u^2 - e^u = x^3$

   (b) $(u_t - 1)^2 - u_t^2 + u_x = 2x$

   (c) $u_x + u_y = 1$

   (d) $2u + 3u_{xt} + 4u_{xxy} = 3x^2 + y$

   (e) $u + u_t + u_x = u^2$

   (f) $\log(u_x) = \log(u_y) + 1$

2. Find the solution of $u_x + yu_y = 0$ on $\mathbb{R}^2$ that satisfies $u(0, y) = y^2$.

3. Find the solution of $u_x + xu_y = 0$ on $\mathbb{R}^2$ that satisfies $u(0, y) = e^y$.

4. Find the general solution of $2u_x + 3u_y = 1$ on $\mathbb{R}^2$.

5. Let $u(x,t) = tx(1-x)\exp\left(\cos(x^2 t)\sin(tx)te^x\right)$. Explain why u cannot be a solution of the heat equation on the rectangular strip $0 \le x \le 1$ and $0 < t < 1$.

6. Solve $u_{xx} + u_{xt} - 6u_{tt} = 0$ with $u(x, 0) = x$ and $u_t(x, 0) = 0$ by factoring the PDE into two transport equations.

7. Solve $u_{xx} - u_{xt} - 12u_{tt} = 0$ with $u(x, 0) = 0$ and $u_t(x, 0) = x$ by factoring the PDE into two transport equations.

8. Solve the wave equation $u_{tt} - u_{xx} = 0$ on the entire real line with initial data $u(x, 0) = \sin(x)$ and $u_t(x, 0) = \cos(x)$.

9. Solve the wave equation $u_{tt} - u_{xx} = 0$ on the entire real line with initial data $u(x, 0) = x^2$ and $u_t(x, 0) = x$.

10. Find a formula for the solution of the heat equation $u_t - u_{xx} = 0$ on the half line $0 < x < \infty$ with initial condition $u(x, 0) = x^2 + 1$ and boundary condition $u(0, t) = 1$ for all $t > 0$.

11. Solve the heat equation $u_t - u_{xx} = 0$ on the entire real line with initial condition $u(x,0) = x$. Use your solution to find the value of the integral

$$\int_{-\infty}^{\infty} xe^{-(x-a)^2}\,dx \quad \text{for } a \in \mathbb{R}.$$

12. Solve the heat equation $u_t - u_{xx} = 0$ on the infinite strip $0 < x < \pi$ and $t > 0$ with initial condition $u(x,0) = \varphi(x) = 1$ and Dirichlet boundary conditions $u(0,t) = u(\pi,t) = 0$ for $t > 0$.

13. Solve the heat equation $u_t - u_{xx} = 0$ on the infinite strip $0 < x < \pi$ and $t > 0$ with initial condition $u(x,0) = \varphi(x) = x$ and Dirichlet boundary conditions $u(0,t) = u(\pi,t) = 0$ for $t > 0$.

14. Let

$$u(x,t) = \begin{cases} (1-t^2)\cos(\exp(\sin(tx^2 - 4x)t^2))e^{-x^2}, & \text{if } 0 \le t \le 1 \\ 0, & \text{if } t \ge 1. \end{cases}$$

Explain why u cannot be a solution of the wave equation on the entire real line.

15. Use separation of variables to find a family of solutions of $u_{tx} - u_{xx} = 0$ on the halfline $x > 0$ and $t > 0$ satisfying the boundary condition $u(0,t) = 0$ for $t > 0$.

16. Show that the sequence of functions $f_n(x) = x^n$ on the interval $(0,1)$ converges to zero in the L^2 sense. Does it converge to zero uniformly? Why or why not?