Math 1272: Calculus II
Introduction

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Overview

In Calculus I (1271), you learned about the **derivative**

$$y'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h},$$

the **integral**

$$\text{(Indefinite)} \int y(x)\,dx \quad \text{and} \quad \text{(Definite)} \int_a^b y(x)\,dx,$$

and the **Fundamental Theorem of Calculus**:

- $\int f'(x)\,dx = f(x) + \text{Constant}$
- $\frac{d}{dx} \int_a^x f(x)\,dx = f(x)$
- $\int_a^b f'(x)\,dx = f(b) - f(a)$.

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$$\int \frac{\sec^2 x}{\tan^2 x + 3\tan x + 2}\, dx$$

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2. Use integration techniques to solve differential equations:

   - (Linear) $y''(x) + y'(x) + y(x) = e^x$
   - (Nonlinear) $y'(x)^2 + y(x)y''(x) = \sin(x)$.

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3. Sequences and series

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}.$$

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$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}.$$
4. Beginning introduction to vector calculus:
$$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\|\cos\theta \quad \text{and} \quad \mathbf{x} \times \mathbf{y} = \|\mathbf{x}\|\|\mathbf{y}\|\sin\theta\mathbf{n}.$$

Differential equations

Differential equations are equations of the form

$$\text{(First order)} \quad F(y'(x), y(x), x) = 0.$$

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For example

- (Simple Harmonic Oscillator) $x''(t) + \frac{k}{m}x(t) = 0$.
- (Electric circuit) $Ri(t) + Li'(t) = V$
- Predator-prey (Lotka-Volterra) equations

$$x'(t) = \alpha x(t) - \beta x(t)y(t) \quad \text{and} \quad y'(t) = \delta x(t)y(t) - \gamma y(t).$$

Differential equations

Differential equations are ubiquitous in physics, largely due to Newton's Law

$$\underbrace{F}_{\text{Force}} = \underbrace{m \times a.}_{\text{Mass}\times\text{Acceleration}}$$

Claim: Newton's Law is a differential equation in disguise!

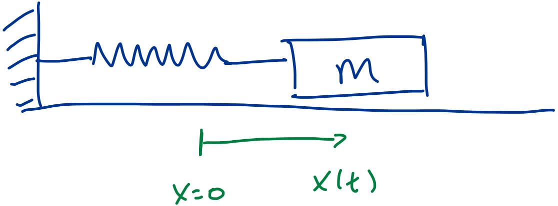
$$a(t) = \frac{dv}{dt} = v'(t) \qquad (v(t) = \text{velocity})$$

$$v(t) = \frac{dx}{dt} = x'(t) \qquad (x(t) = \text{position})$$

$$a(t) = x''(t)$$

$$\boxed{F = m\,x''(t)}$$

Example (Simple harmonic oscillator)



Hooke's Law

$$F = -Kx(t)$$

Newton's Law

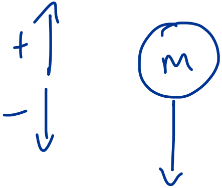
$$F = ma$$

$$-Kx(t) = mx''(t)$$

$$mx''(t) + Kx(t) = 0$$

$$\boxed{x''(t) + \frac{k}{m}x(t) = 0}$$

Example (Falling mass)



Air resistance force

$$F_{air} = -k\,x'(t)$$

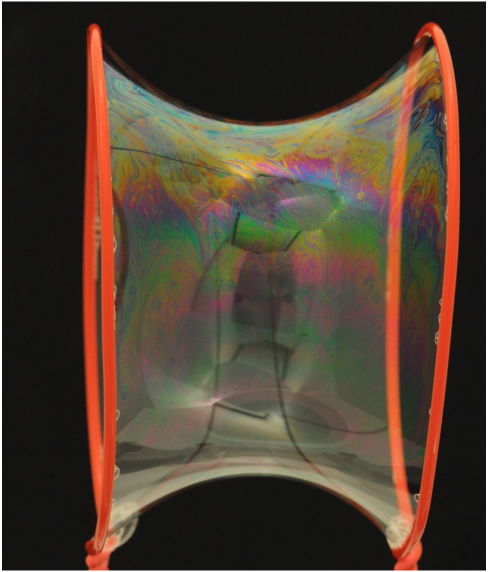
$$F_{gravity} = -mg$$

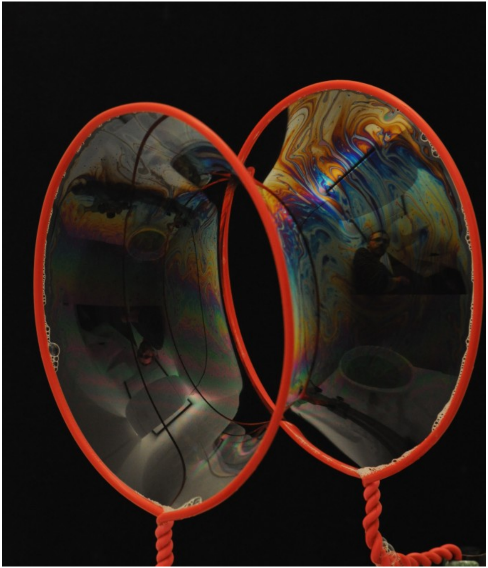
$$F = ma$$

$$-k\,x'(t) - mg = m\,x''(t)$$

$$\boxed{x''(t) + \frac{k}{m}x'(t) + g = 0}$$

Minimal surfaces (https://www.soapbubble.dk/)





Minimal surfaces

A soap bubble takes the shape with the least surface area.

For a surface of revolution with height $y(x)$ over the x-axis, y satisfies

$$c^2 y(x)^2 - y'(x)^2 = 1.$$

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The solution is a **catenoid**

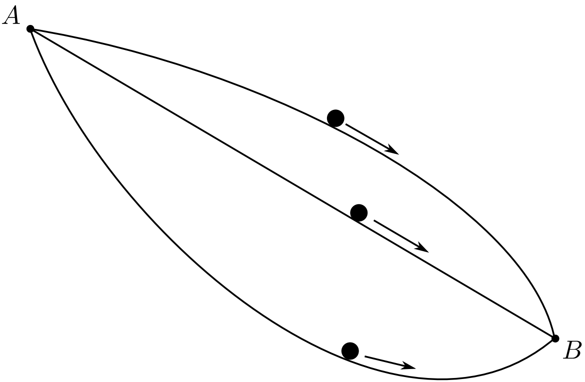
$$y(x) = \frac{e^{cx} + e^{-cx}}{2c}$$

Soap bubble videos:

- https://www.youtube.com/watch?v=3QgHxnDsrGQ?
- https://www.youtube.com/watch?v=ciciWBz8m_Y

Brachistochrone curve

Question: What is the shortest path for a rolling ball?



Brachistochrone demo:

- https://www.youtube.com/watch?v=0KjUqPps8vM

Brachistochrone curve

The Brachistochrone curve $(x, y(x))$ satisfies the differential equation

$$y(x) + y(x)y'(x)^2 = C.$$

The solution is a cycloid (https://en.wikipedia.org/wiki/Cycloid):