Math 1272: Calculus II
9.6 Predator-Prey system

Instructor: Jeff Calder
Office: 538 Vincent
Email: jcalder@umn.edu

http://www-users.math.umn.edu/~jwcalder/1272S19

Predator-Prey (Rabbits/Wolves)

- $R(t)$ = population of prey (rabbits)
- $W(t)$ = population of predator (wolves)

Without iteraction between predator and prey:

$$\frac{dR}{dt} = kR \quad \text{and} \quad \frac{dW}{dt} = -rW.$$

Prey has unlimited resources

Only food source is prey.

Predator-Prey (Rabbits/Wolves)

- We assume the number of "interactions" is proportional to $R(t)W(t)$.
- Each interaction decreases the number of prey, and provides food for the predator, increasing their propulation.

Thus, a more realisic model is

$k, r, a, b > 0$

$$\frac{dR}{dt} = kR - aRW$$
$$\frac{dW}{dt} = -rW + bRW.$$

These are the **Lotka-Volterra equations**. They form a coupled system of differential equations.

Predator-Prey (Rabbits/Wolves)

- We assume the number of “interactions” is proportional to $R(t)W(t)$.
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Even more realistic is to use the **logistic** model for prey

$$\frac{dR}{dt} = kR\left(1 - \frac{R}{M}\right) - aRW$$

$$\frac{dW}{dt} = -rW + bRW.$$

Assume $k = 2$ $r = 1$, $a = 10$ and $b = 5$. The Lotka-Voltera equations are

$$\frac{dR}{dt} = 2R - 10RW = 0$$
$$\frac{dW}{dt} = -W + 5RW. = 0$$

Find the **equillibrium solutions**.

$2R - 10RW = 0 \qquad 2 = 10W, \quad W = \frac{2}{10} = \frac{1}{5}$

$-W + 5RW = 0, \quad 5R = 1, \quad R = \frac{1}{5}$

$(W, R) = \left(\frac{1}{5}, \frac{1}{5}\right)$

$$\frac{dR}{dt} = 2R - 10RW$$
$$\frac{dW}{dt} = -W + 5RW.$$

$$\frac{dR}{dt} = \frac{1}{\frac{dt}{dR}}$$

Find an equation for $\frac{dW}{dR}$.

$$\frac{dw}{dR} = \frac{dw}{dt} \cdot \frac{dt}{dR} = \frac{\frac{dw}{dt}}{\frac{dR}{dt}}$$

$$\frac{dw}{dR} = \frac{-w + 5Rw}{2R - 10Rw}$$

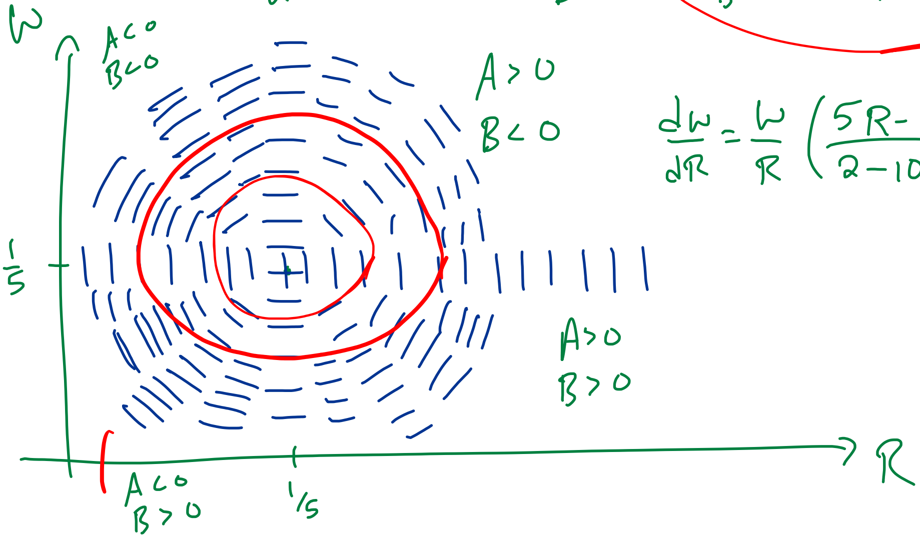
Sketch the direction field for $\frac{dW}{dR}$, and some solutions.

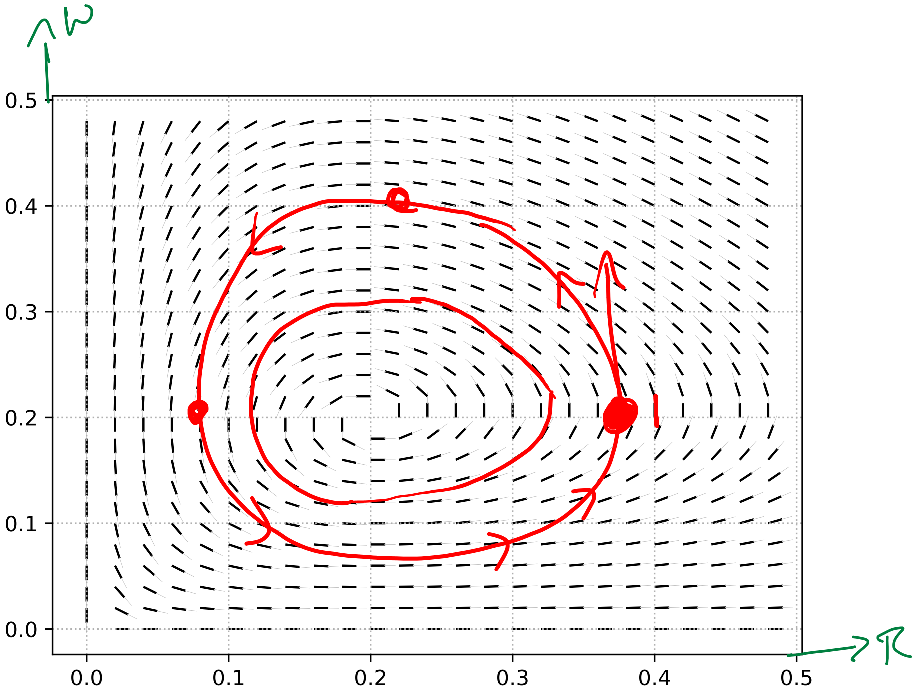
$$\frac{dw}{dR} = \frac{-w + 5Rw}{2R - 10Rw} = \frac{A}{B}$$

$A = 0$ if $R = \frac{1}{5}$

$B = 0$ if $w = \frac{1}{5}$

$$\frac{dw}{dR} = \frac{w}{R}\left(\frac{5R - 1}{2 - 10w}\right)$$





w
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.1
0.2
0.3
0.4
0.5
R