Math 222A – Homework 1 (Due Friday Sept 4)

1. Do all the exercises in Section 1.5 of Evans.

2. Let k be a positive integer. Show that a smooth function defined on $\mathbb{R}^n$ has in general

$$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

distinct partial derivatives of order k. [1]

3. Evans: Section 2.5, Problem 1

4. Evans: Section 2.5, Problem 2

5. Let u denote the density of particles moving to the right with speed one along the real line and let v denote the density of particles moving to the left with speed one. If at rate $d > 0$ right-moving particles randomly become left-moving, and vice versa, we have a system of PDE

$$\begin{cases} u_t + u_x = d(v-u) \\ v_t - v_x = d(u-v). \end{cases}$$

Show that both $w := u$ and $w := v$ solve the telegraph equation

$$w_{tt} + 2dw_t - w_{xx} = 0.$$ [2]

6. Evans: Section 2.5, Problem 21 (Problem 16 in 1st edition)

[1] Section 1.5, Problem 2 in 2nd edition
[2] Section 2.5, Problem 22 in 2nd edition