Math 5587 Midterm I Information

- The midterm will take place on Thursday, October 6, during class.
- The exam will cover everything up to and including the lecture on Thurs September 22.
- The exam is closed book. No textbooks, notes, or calculators are allowed.
- The exam will have 5 questions. The first 3 will be short, and the last 2 will be longer and slightly more involved. Below are a collection of sample midterm questions for you to practice.

Sample questions

1. For each of the following equations state whether it is nonlinear, linear homogeneous, or linear inhomogeneous.

   (a) $u_{ttt} + u^2 - e^u = x^3$

   (b) $(u_t - 1)^2 - u_t^2 + u_x = 2x$

   (c) $u_x + u_y = 1$

   (d) $2u + 3u_{xt} + 4u_{xxy} = 3x^2 + y$

   (e) $u + u_t + u_x = u^2$

   (f) $\log(u_x) = \log(u_y) + 1$

2. Determine whether the following statements are true or false. No justification is required.

   (a) A PDE of the form $L(u) = F(u, u_x, u_y, u_{xx}, u_{xy}, u_{yy}) = 0$ is linear provided
   $$L(au + v) = aL(u) + L(v)$$
   for all real numbers a and functions $u(x, y)$ and $v(x, y)$.

   (b) If $u(x, t)$ is a solution of the heat equation on the rectangle $0 \le x \le 1$ and $0 \le t \le 1$, then the maximum of u over the rectangle must be attained on the base $0 \le x \le 1$ and $t = 0$ of the rectangle.

   (c) Let $u(x, t)$ be the solution of the wave equation on the entire real line $-\infty < x < \infty$ with initial position $u(x, 0) = f(x)$ and initial velocity $u_t(x, 0) \equiv 0$. If $f(x) = 0$ for all $|x| \ge 1$, then $\lim_{t\to\infty} u(x, t) = 0$ for every x.

3. Find the solution of $u_t + xu_x = 0$ that satisfies $u(x, 0) = x^2$.

4. Find the solution of $u_t + tu_x = 0$ on $\mathbb{R}^2$ that satisfies $u(x, 0) = e^x$.

5. Find the general solution of $2u_x + 3u_y = 1$ on $\mathbb{R}^2$.

6. Find the solution of the linear PDE $xu_t - tu_x = 0$ on the domain $x > 0$ and $t \ge 0$ that satisfies $u(x, 0) = x^2$ for all $x \ge 0$.

7. Let $u(x, t) = tx(1 - x) \exp\left(\cos(x^2 t) \sin(tx) te^x\right)$. Explain why u cannot be a solution of the heat equation on the rectangular strip $0 \le x \le 1$ and $0 < t < 1$.

8. Solve $u_{xx} + u_{xt} - 6u_{tt} = 0$ with $u(x,0) = x$ and $u_t(x,0) = 0$ by factoring the PDE into two transport equations.

9. Solve $u_{xx} - u_{xt} - 12u_{tt} = 0$ with $u(x,0) = 0$ and $u_t(x,0) = x$ by factoring the PDE into two transport equations.

10. Solve the wave equation $u_{tt} - u_{xx} = 0$ on the entire real line with initial data $u(x,0) = \sin(x)$ and $u_t(x,0) = \cos(x)$.

11. Solve the wave equation $u_{tt} - u_{xx} = 0$ on the entire real line with initial data $u(x,0) = x^2$ and $u_t(x,0) = x$.

12. Solve the wave equation $u_{tt} - u_{xx} = 0$ on the entire real line $-\infty < x < \infty$ with initial position $u(x,0) \equiv 0$ and initial velocity $u_t(x,0) = \frac{4x}{x^2+1}$.

13. Find a formula for the solution of the heat equation $u_t - u_{xx} = 0$ on the half line $0 < x < \infty$ with initial condition $u(x,0) = x^2 + 1$ and boundary condition $u(0,t) = 1$ for all $t > 0$.

14. Solve the heat equation $u_t - u_{xx} = 0$ on the entire real line with initial condition $u(x,0) = x$. Use your solution to find the value of the integral

$$\int_{-\infty}^{\infty} x e^{-(x-a)^2}\, dx \quad \text{for } a \in \mathbb{R}.$$

15. Let

$$u(x,t) = \begin{cases} (1-t^2)\cos(\exp(\sin(tx^2 - 4x)t^2))e^{-x^2}, & \text{if } 0 \le t \le 1 \\ 0, & \text{if } t \ge 1. \end{cases}$$

Explain why u cannot be a solution of the wave equation on the entire real line.