Computer Vision

Programming a computer to interpret images or videos in a meaningful way.
Overview of Computer Vision

- **Image Acquisition**
  - Single-pixel camera
  - Compressed sensing

- **Pre-processing**
  - Noise reduction
  - Resampling
  - Contrast enhancement

- **Mid-level Processing**
  - Detection
  - Segmentation
  - Feature extraction

- **High-level Processing**
  - Object recognition
  - Classification

- **Decision**
Overview of Computer Vision

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- **Pre-processing**
  - Noise reduction, resampling
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- **Decision**
Basics

An grayscale image is a function $u : [0, 1]^2 \rightarrow [0, 1]$.

$$u(x) = \text{Pixel Intensity at position } x.$$
Image Segmentation

Partition an image into meaningful segments.

(a) Lenna image  
(b) Possible segmentation
Notation

\[ C : s \mapsto \mathbf{x}(s) = (x_1(s), x_2(s)) \]

\[ \hat{n} = (x_2', x_1') \]

\( C = \) Segmentation boundary \quad \Omega_1 = \) Segmented region \quad \Omega_2 = \) Background.
Basic segmentation model

Assumption:
The image $u$ is piecewise constant with two regions and is corrupted by noise.
Basic segmentation model

Consider the following energy functional

\[ E(C, c_1, c_2) = \int_{\Omega_1} |u(x, y) - c_1|^2 \, dx \, dy + \int_{\Omega_2} |u(x, y) - c_2|^2 \, dx \, dy + \lambda \oint_C ds. \]
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- The segmentation \( C \) is obtained by minimizing \( E \) over \( C, c_1 \) and \( c_2 \).
  - If \( C \) is fixed and we minimize over only \( c_1 \) and \( c_2 \) we get

\[ c_i = \text{Average of } u \text{ in } \Omega_i. \]
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  - Special case of Mumford-Shah functional
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  - Special case of Mumford-Shah functional

Question: How do we go about minimizing \( E \)?
Functions on $\mathbb{R}^n$

Suppose we want to minimize a real valued function on $\mathbb{R}^n$

$$f : \mathbb{R}^n \to \mathbb{R}$$
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$$f : \mathbb{R}^n \to \mathbb{R}$$

Idea: Start at a point $x \in \mathbb{R}^n$ and move in a direction $v$ that decreases $f$ as quickly as possible:

$$\left. \frac{d}{dt} \right|_{t=0} f(x + tv) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(x)v_i = \nabla f(x) \cdot v$$

Direction of steepest descent is $v = -\nabla f(x)$.

Gradient Descent:

$$x(0) = x_0, \quad \frac{dx}{dt}(t) = -\nabla f(x(t))$$
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Minimizing the length of a curve

Let \( x : [a, b] \to \mathbb{R}^2 \) be a curve in \( \mathbb{R}^2 \).

Question: In which direction should we move the curve to decrease its length as fast as possible? Enough to consider only normal perturbations \( v(s) = \hat{n}(s) v(s) \), for some \( v : [a, b] \to \mathbb{R}^2 \).
Minimizing the length of a curve

Let \( x : [a, b] \rightarrow \mathbb{R}^2 \) be a curve in \( \mathbb{R}^2 \)

**Question:** In which direction \( v : [a, b] \rightarrow \mathbb{R}^2 \) should we move the curve to decrease its length as fast as possible?
Minimizing the length of a curve

Let $\mathbf{x} : [a, b] \rightarrow \mathbb{R}^2$ be a curve in $\mathbb{R}^2$

**Question:** In which direction $\mathbf{v} : [a, b] \rightarrow \mathbb{R}^2$ should we move the curve to decrease its length as fast as possible?

Enough to consider only normal perturbations

$$\mathbf{v}(s) = \hat{n}(s)\cdot v(s),$$

for some $v : [a, b] \rightarrow \mathbb{R}$. 
Let's assume $x$ is parameterized by arclength ($\|x'(s)\| = 1$) so that

$$\text{Length of } x = L(x) = \int_a^b \|x'(s)\| \, ds = b - a.$$
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Consider the directional derivative

$$\left. \frac{d}{dt} \right|_{t=0} L(x + tv) = \left. \frac{d}{dt} \right|_{t=0} \int_a^b \|x'(s) + tv'(s)\| \, ds$$
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$$
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\]

\[
= \int_a^b \frac{d}{dt} \bigg|_{t=0} \|y(t)\| = \frac{y(t) \cdot \frac{d}{dt} y(t)}{\|y(t)\|}
\]

\[
= \int_a^b x' \cdot (\hat{n}'v + \hat{n}v') \, ds
\]
Let’s assume \( x \) is parameterized by arclength \((\| x'(s) \| = 1)\) so that

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Consider the directional derivative

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\[
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Consider the directional derivative

\[
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\]

\[
= \int_a^b \frac{d}{dt}\bigg|_{t=0} \|x' + t(\hat{n}' + \hat{n}v')\| \, ds
\]

\[
= \int_a^b (x' \cdot \hat{n}') v \, ds = \langle x' \cdot \hat{n}', v \rangle L^2(a,b).
\]
Let's assume $x$ is parameterized by arclength ($\|x'(s)\| = 1$) so that

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$$\left. \frac{d}{dt} \right|_{t=0} L(x + tv) = \left. \frac{d}{dt} \right|_{t=0} \int_a^b \|x'(s) + tv'(s)\| \, ds$$

$$= \int_a^b \left. \frac{d}{dt} \right|_{t=0} \|x' + t(\hat{n}'v + \hat{n}v')\| \, ds$$

$$= \int_a^b x' \cdot (\hat{n}'v + \hat{n}v') \, ds$$

$$= \int_a^b (x' \cdot \hat{n}') v \, ds = \langle x' \cdot \hat{n}', v \rangle_{L^2(a,b)}.$$ 

Choosing $v = -x' \cdot \hat{n}'$ we have

$$\left. \frac{d}{dt} \right|_{t=0} L(x + tv) = -\|x' \cdot \hat{n}\|^2_{L^2(a,b)}.$$
Curvature motion

Recalling that $\hat{n} = (-x_2', x_1')$ we have

$$v = -x' \cdot \hat{n}'$$
Curvature motion

Recalling that \( \hat{n} = (-x_2', x_1') \) we have

\[
\begin{align*}
v & = -x' \cdot \hat{n}' \\
& = -(x_1', x_2') \cdot (-x_2'', x_1'')
\end{align*}
\]
Curvature motion

Recalling that \( \hat{n} = (-x'_2, x'_1) \) we have

\[
v = -x' \cdot \hat{n}' = -(x'_1, x'_2) \cdot (-x''_2, x''_1) = x'_1 x''_2 - x'_2 x''_1
\]

Answer: To decrease the length of \( x \) as fast as possible, we should move in the direction \( v = \kappa \hat{n} \).

This is called Motion by Curvature:

\[
x(0, s) = x(0, s), \quad \frac{\partial x}{\partial t}(t, s) = \kappa(t, s) \hat{n}(t, s)
\]
Curvature motion

Recalling that $\hat{n} = (-x_2', x_1')$ we have

\[
v = -\mathbf{x}' \cdot \hat{n}' \\
  = -(x_1', x_2') \cdot (-x_2'', x_1'') \\
  = x_1' x_2'' - x_2' x_1'' \\
  = \kappa \quad \text{Curvature!}
\]
Curvature motion

Recalling that $\hat{n} = (-x_2', x_1')$ we have

$$v = -\mathbf{x}' \cdot \hat{n}' = -(x_1', x_2') \cdot (-x_2'', x_1'') = x_1' x_2'' - x_2' x_1'' = \kappa \text{ Curvature!}$$

Answer: To decrease the length of $\mathbf{x}$ as fast as possible, we should move in the direction $\mathbf{v} = \kappa \hat{n}$. 
Curvature motion

Recalling that $\hat{n} = (-x_2', x_1')$ we have

\[ v = -x' \cdot \hat{n}' \]
\[ = -(x_1', x_2') \cdot (-x_2'', x_1'') \]
\[ = x_1' x_2'' - x_2' x_1'' \]
\[ = \kappa \quad \text{Curvature!} \]

Answer: To decrease the length of $x$ as fast as possible, we should move in the direction $v = \kappa \hat{n}$.

This is called **Motion by Curvature**:

\[
\begin{align*}
  x(0, s) &= x_0(s), \\
  \frac{\partial x}{\partial t}(t, s) &= \kappa(t, s)\hat{n}(t, s).
\end{align*}
\]
Curvature motion demo 1
Curvature motion demo 2
Gradient descent on $E$

$$E(C, c_1, c_2) = \int_{\Omega_1} |u(x, y) - c_1|^2 \, dxdy + \int_{\Omega_2} |u(x, y) - c_2|^2 \, dxdy + \lambda \oint_C ds.$$ 

Following a similar procedure, the gradient descent equation for $E$ is

$$\begin{cases} 
\frac{\partial x}{\partial t} = (|u(x) - c_1|^2 - |u(x) - c_2|^2 + \lambda \kappa) \hat{n} \\
\frac{1}{|\Omega_i|} \int_{\Omega_i} u(x, y) \, dxdy \\
x(0, \cdot) = x_0.
\end{cases}$$
(a) $\lambda = 0$

(b) $\lambda = 0.15$

(c) $\lambda = 0.5$

$$E(C, c_1, c_2) = \int_{\Omega_1} |u(x, y) - c_1|^2 \, dxdy + \int_{\Omega_2} |u(x, y) - c_2|^2 \, dxdy + \lambda \oint_C ds.$$
Segmentation demo 2
Medical imaging demo 1
Medical imaging demo 2
Medical imaging demo 3

![CT Scan Image]

[Red outline indicates a specific area of interest]
Questions?

Some resources:

- **Image Processing On Line**: [http://www.ipol.im](http://www.ipol.im)
  - C++ code and demos for segmentation, denoising, deblurring, etc.

- **CImg Library**: [http://cimg.sourceforge.net](http://cimg.sourceforge.net)
  - C++ template for image processing.