HOTT EXERCISE 5.1

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Problem 5.1 Derive the induction principle for the type \( \text{List}(A) \) of lists from its definition as an inductive type in §5.1.

Proof. When proving a statement \( E : \text{List}(A) \to U \) about all lists over a type \( A \), it suffices to prove it for \( \text{nil} : \text{List}(A) \) and \( \text{cons} : (a,l) \), assuming it holds true for \( l \) and that \( a : A \).

We construct \( e_n : E(\text{nil}) \) and \( e_s : \prod_{a:A} \prod_{l:\text{List}(A)} E(l) \to E(\text{cons}(a,l)) \). This yields the computation rules for \( \text{ind}_{\text{List}(A)}(E, e_n, e_s) : \prod_{l:\text{List}(A)} E(l) \).

- \( \text{ind}_{\text{List}(A)}(E, e_n, e_s, \text{nil}) \equiv e_n \)
- \( \text{ind}_{\text{List}(A)}(E, e_n, e_s, \text{cons}(a,l)) \equiv e_s(l, \text{ind}_{\text{List}(A)}(E, e_n, e_s, l)) \)

\( \square \)