From the Hubbard Text:
Page 342-343: 2, 4, 5, 10, 12, 15, 16
Page 350-351: 1, 2, 3, 8.

Additional Problems:
1) find $\nabla f(x)$ for each of the following functions:
   a) $f(x) = x_0 \cdot x$
   b) $f(x) = |x|, x \neq 0$
   c) $f(x) = (x_0 \cdot x)^2$
2) (Euler’s formula): Let $p$ be a real number. A function $f$ is called homogeneous of degree $p$ if
   
   \[ f(tx) = t^p f(x), \]

   for every $x \neq 0, t > 0$. Let $f$ be a smooth real-valued function on $\mathbb{R}^n$.
   Show that $f$ is homogeneous of degree $p$ if and only if
   
   \[ Df(x) \cdot x = pf(x) \]

   for every $x \neq 0$.
   (Hint: Let $\phi(t) \equiv f(tx)$ and differentiate this with respect to $t$, using the chain rule. For the converse, show that for fixed $x$, the function $\phi(t) t^{-p}$ is a constant.
3) Let $Q(x) = \sum_{i,j=1}^n C_{ij} x_i x_j$, where the $x_i$ are the coordinates of $x$ and $C_{ij} = C_{ji}$. Assume also $Q(x) > 0$ for all $x \neq 0$. Let
   
   \[ f(x) = [Q(x)]^{\frac{1}{2}} \]

   Calculate $Df(x)$ and verify Euler’s formula for this function.
4) Let $x_0$ be a nondegenerate critical point of a function $f$ which is smooth, $f: \mathbb{R}^n \to \mathbb{R}$.
   Show that $x_0$ is isolated. That is, show that there is an open set $U$ with $x_0 \in U$ and so that $U$ contains no other critical values of $f$ other than $x_0$.
   (Hint: Let $x$ be another critical point in $U$. Apply the mean value theorem to each of the functions $f_1, f_2, \ldots, f_n$ where $f_i = \frac{\partial f}{\partial x_i}$ to find that,

   \[ 0 = \sum_{j=1}^n f_{ij} (y_j) (x^j - x_0^j), \quad i = 1, \ldots, n, \]

   where each $y_j \in U$. You may assume that if a matrix whose entries are continuous functions is invertible at a point, then it is invertible in some sufficiently small neighborhood of the point (we’ll prove this later). Conclude that (1) can only hold for $x - x_0 = 0$, a contradiction.)