MATH 3283W
Fall 2017
Exam 3
Thursday 30 November 2017
Time Limit: 50 minutes

Name (Print): ____________________
Student ID: ____________________
Section Number: ______
Teaching Assistant: ____________________
Signature: ____________________

This exam contains 5 numbered problems on seven sheets of paper. The last sheet is blank. Check to see if any pages are missing. Point values are in parentheses. No books, notes, or electronic devices are allowed.

As on the writing quizzes, your work will be graded on the quality of your writing as well as on the validity of the mathematics. In particular, included in the 20 points for problem 3 is a five-point writing score.

Do not use symbols for logical connectives and quantifiers. That is, do not use the symbols $\Rightarrow$, $\Leftrightarrow$, $\wedge$, $\vee$, $\sim$, $\forall$, $\exists$, and $\ni$.

1	20 pts	
2	20 pts	
3	20 pts	
4	10 pts	
5	20 pts	
TOTAL	90 pts	

1. (20 points) Are the following three statements True or False? If you believe a statement is False, you must start your answer with the word "False" and then give a counterexample (and explain why it is a counterexample). If you believe a statement is True, start your answer with the word "True" and then give a proof. (Your proofs can use any result in the textbook reading or any result we covered in the course lectures.)

   (a) (7 points)True or False: If the set $S \subset \mathbf{R}$ is compact and x_0 is an accumulation point for S, then $x_0 \in S$.

   (b) (7 points) True or False: Some unbounded sets in $\mathbf{R}$ are compact.

   (c) (6 points) True or False: If a set has a maximum and a minimum then the set is compact.

2. (20 points) For s_j given by the following formulae, determine the convergence or divergence of the sequence $(s_j)_{j=1}^{\infty}$. Find any limits that exist. (So for example, if the sequence diverges, and also goes to ∞, you should say this.) Show all work and explain your reasoning. You may use any results from the reading or discussed in the lectures.

a) [5 points]

$$s_j = \left(\frac{1}{j}\right)^{\frac{1}{j}}$$

.

b). [7 points]

$$s_j = \frac{j^j}{(2j)!}.$$

c) [8 points]

$$s_j = \sqrt{j^2 + j} - j$$

.

3. (20 points) Define a sequence $(s_n)_{n=1}^{\infty}$ as follows: $s_1 = \sqrt{5}$, $s_2 = \sqrt{5+\sqrt{5}}$, $s_3 = \sqrt{5+\sqrt{5+\sqrt{5}}}$, and in general define $s_{n+1} = \sqrt{5+s_n}$. Prove carefully (making it clear and explicit if you use any results discussed in the reading or in the lectures) that the sequence converges and find its limit. The question is worth 15 points, with an additional 5 points assigned as a writing score for this problem.

4. (10 points) Determine the limit of the sequence (s_n) with terms,

$$s_n = \frac{2n+3}{n^2-13}.$$

Justify your answer directly from the definition of convergence. Do not use any theorems that have been proven in class or in the textbook.

5. (20 points) (a) (3 points) Given any bounded sequence $(u_n)_{n=1}^{\infty}$, define what we mean by $\liminf u_n$ and $\limsup u_n$.

(b). (2 points) Consider the sequence $(u_n)_{n=1}^{\infty}$ where,

$$u_n = n^2(-1 + (-1)^n),$$

and state what values the two quantities $\liminf u_n$ and $\limsup u_n$ have for this particular example.

(c) (15 points). Let $(a_n)_{n=1}^{\infty}$ be a sequence of real numbers which is bounded. Let $L = \limsup a_n$. Prove that there is a subsequence $(a_{n_j})_{j=1}^{\infty}$ so that $\lim_{j\to\infty} a_{n_j} = L$. You may use any result discussed in class or the reading in our textbook in your proof.

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