

MATH 3283W
Spring 2017
Exam 3
Thursday 13 April 2017
Time Limit: 50 minutes

Name (Print): _____
Student ID: _____
Section Number: _____
Teaching Assistant: _____
Signature: _____

This exam contains 5 numbered problems on seven sheets of paper. The last sheet is blank. Check to see if any pages are missing. Point values are in parentheses. No books, notes, or electronic devices are allowed.

As on the writing quizzes, your work will be graded on the quality of your writing as well as on the validity of the mathematics. In particular, a five-point writing score (W) will be determined from your work on problems 1, 2c, 3b, 4, and 5b.

Do not use symbols for logical connectives and quantifiers. That is, do not use the symbols \Rightarrow , \Leftrightarrow , \wedge , \vee , \sim , \forall , \exists , and \ni .

1	10 pts	
2	12 pts	
3	13 pts	
4	7 pts	
5	10 pts	
W	5 pts	
TOTAL	57 pts	

1. (10 points) Determine the limit of the sequence $\{s_n\}$ with terms

$$s_n = \frac{n^2 - 5}{2n - n^3}.$$

Justify your answer directly from the definition of convergence. Do not use any theorems that have been proven in class or in the textbook.

2. (12 points) (a) (3 points) Give an example of a non-empty, closed, bounded subset S of \mathbb{R} for which $\sup S$, the least upper bound of S , is an accumulation point of S . Briefly justify your answer.

(b) (3 points) Give an example of a non-empty, closed, bounded subset S of \mathbb{R} for which $\sup S$ is **NOT** an accumulation point of S . Briefly justify your answer.

(c) (6 points) Prove that every non-empty, closed subset S with an upper bound has a maximum by showing that $\sup(S)$ belongs to S . (Hint: Consider cases based on the above parts.)
This was an important lemma used to prove the Heine-Borel theorem.

3. (13 points) (a) (3 points) State the Monotone Convergence Theorem for the real numbers \mathbb{R} .

(b) (10 points) Define the sequence $\{a_n\}$ by $a_1 = 1$ and $a_{n+1} = \sqrt{2a_n}$. Show that $\{a_n\}$ converges and find its limit.

4. (7 points) Let $\{s_n\}$ be given by the formula

$$s_n = \begin{cases} \frac{n}{n+1} & \text{if } n \text{ is odd,} \\ \frac{n^2}{n+1} & \text{if } n \text{ is even.} \end{cases}$$

Find $\limsup s_n$ and $\liminf s_n$. Be sure to justify your answer.

5. (10 points) (a) (3 points) Complete the following definition: A sequence $\{s_n\}$ of real numbers is said to be a **Cauchy sequence** if ...

(b) (7 points) Prove that every convergent sequence is a Cauchy sequence.

(blank)