

MATH 3283W

Gateway Quiz 1: 15 minutes to complete. This quiz is closed books, closed notes, no phones or electronic devices allowed.

Tuesday 9-19-17

Name Solutions

Remember, your work will be graded on the quality of your writing as well as on the validity of the mathematics. The quiz is worth 20 points, five of which are for a writing score.

Do not use symbols for logical connectives and quantifiers. That is, do not use the symbols \Rightarrow , \Leftrightarrow , \wedge , \vee , \sim , \forall , \exists , and \ni .

This quiz concerns the following two statements about real numbers x , y , and z . One of these statements is true, and one of the statements is false.

A. There exists y so that for all x , $xy = 1$.

B. There exists x so that for all y and z , $z > y$ implies that $z > x + y$.

1. (7 points) Identify clearly which statement is true (write "A is true" or "B is true"). After you write this, prove that statement which you have identified as true.

B is true.

Proof of B. Let $x=0$. Then $y = y+0 = y+x$, for any y .
Therefore, for any z and y , if $z > y$, then $z > x+y$. \square

Caution: x cannot depend on y or z , because of the order of the quantifiers. So you couldn't say something like $x = z - y$ or anything like this!

2. (8 points) Write the negation of whichever of the two statements above is false. Write this without using symbols for logical connectives and quantifiers. After you have written this negation, write a proof of this negation of the false statement.

$\sim A$: For all y , there exists x such that $xy \neq 1$.

Proof of $\sim A$: Fix y . Then note that if $x=0$, then

$$xy = (0)y = 0 \neq 1. \text{ can be done here.}$$

Hence ~~for~~ for any y , there exists x s.t. $xy \neq 1$, namely, $x=0$. \square

Remark: Actually, since $x=0$ works for all y , the stronger statement $\exists x$ s.t. $\forall y, xy \neq 1$, is true. \square

