Math 4242: Linear Algebra
MidtermII, November 16 2016

Directions:

PLEASE DO NOT OPEN EXAM UNTIL DIRECTED TO DO SO.
This is a closed book exam. No books. No notes. No crib sheets. No calculators.

You are allowed 50 minutes to complete this exam.

Please show all your work on the enclosed pages. You are not allowed any scratch paper of your own.

There are 7 questions. Including this title page, there are 11 pages (the last two of which are blank).

Please make sure all the pages are here before beginning your 50 minutes of work.

Scores:

(1) (5 pts) ______________

(2) (4 pts) ______________

(3) (4 pts) ______________

(4) (6 pts) ______________

(5) (5 pts) ______________

(1) (5 points) Find the dimensions of the four fundamental subspaces associated with the matrix,

$$A = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 4 & 1 & 3 \\ 3 & 6 & 1 & 4 \end{pmatrix}$$

Show all work.

(2) (4 points) Let P_2, P_3 be the spaces of polynomials (with real coefficients) of degrees less than or equal to 2 and 3, respectively. Consider the map $S : P_2 \to P_3$ defined by,

$$S(p) = \int_0^t p(x)dx$$

Determine the matrix $[S]_{B\tilde{B}}$ where

$$B = \{1 + \frac{1}{2}t, t, \frac{1}{3}t^2\}$$

$$\tilde{B} = \{1, t, t^2, \frac{1}{4}t^3\}$$

(3) (4 points) Suppose that A and B are two matrices which are similar. Explain why it's true that A^k and B^k are also similar for any nonnegative integer k. (Here A^k represents the kth power of A, similarly B^k.)

(4) (6 points) **This problem covers the next two pages of your exam.**

Suppose Q is a 4 by 3 matrix with **orthonormal** columns $\vec{q}_1, \vec{q}_2, \vec{q}_3$.

(a). [2 points] Suppose $\vec{v}$ is a vector in R^4 **NOT** in the range of Q. If we start with the following list of vectors:

$$\{\vec{q}_1, \vec{q}_2, \vec{q}_3, \vec{v}\}$$

and apply the Gram-Schmidt procedure, we will get a list of four orthonormal vectors. The first three vectors in our new orthonormal list will just be $\vec{q}_1, \vec{q}_2, \vec{q}_3$. What will be the fourth vector in the orthonormal list?

ANSWER: $\vec{q}_4 =$ ________________________________

(b). (Multiple Choice - 2 points each) Using the answers on the bottom of this page, fill in the following blanks. You should give at least one sentence justifying each answer.(No credit given for any answer given without justification.)

The nullspace of Q is ____________.

The nullspace of Q^T is ____________.

Your choices for these questions are: (You may use a choice more than once.)

(a). span$\{\vec{q}_1\}$	(b). span$\{\vec{q}_2\}$	(c). span$\{\vec{q}_3\}$	(d). span$\{\vec{q}_4\}$
(e). $\vec{0}$	(f). span$\{\vec{q}_1, \vec{q}_4\}$	(g). span$\{\vec{q}_1, \vec{q}_3\}$	(h). span$\{\vec{q}_1, \vec{q}_2\}$
(i). R^3	(j). R^4	(k). span$\{\vec{v}\}$	(l). span$\{\vec{q}, \vec{v}\}$

(5) . [5 points] Suppose that,

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

is an $n \times n$ matrix such that $\mathrm{Null}(A_1) = \mathrm{Range}(A_2^T)$. (Here and in the definition of A, both A_1 and A_2 are themselves matrices - that is, we have written A in block matrix form.)

Explain carefully why A must be nonsingular.