Math 4242; Spring 2018; Quiz 1: 15 minutes to complete.
Monday, January 29, 2018

No books, no notes, no calculators (unless approved specifically by the instructor).

(1) (4 points) Solve the following system of equations using Gaussian elimination and back substitution. Show all work. (You can use Gauss-Jordan elimination if you prefer).

$$\begin{aligned} 2x + 8y + 4z &= 2 \\ 2x + 5y + z &= 5 \\ 4x + 10y - z &= 1 \end{aligned}$$

Solution. We'll solve the give system using Gauss Jordan elimination. For clarity, we will label the top, middle, and bottom equations as (1),(2), and (3), respectively. Finally, before diving into the solution, we'll use the convention $a(x) + (y)$ to denote adding a times equation x plus equation y.

$$\begin{aligned} (3) + (2) \rightarrow 2x + 8y + 4z &= 2 \\ 6x + 15y + 0 &= 6 \\ 4x + 10y - z &= 1 \end{aligned}$$

$$\begin{aligned} 4(3) + (1) \rightarrow 18x + 48y + 0 &= 6 \\ 6x + 15y + 0 &= 6 \\ 4x + 10y - z &= 1 \end{aligned}$$

$$\begin{aligned} -\frac{2}{3}(2) + (3) \rightarrow 18x + 48y + 0 &= 6 \\ 6x + 15y + 0 &= 6 \\ 0 + 0 - z &= 3 \end{aligned}$$

$$\begin{aligned} -3(2) + (1) \rightarrow 0 + 3y + 0 &= -12 \\ 6x + 15y + 0 &= 6 \\ 0 + 0 - z &= 3 \end{aligned}$$

$$\begin{aligned} -5(1) + (2) \rightarrow 0 + 3y + 0 &= -12 \\ 6x + 0 + 0 &= 66 \\ 0 + 0 - z &= 3. \end{aligned}$$

Finally, putting the reduced system in Gauss Jordan form yields the solution,

$$x = 11 \quad y = -4 \quad z = 3.$$

□

(2) (6 points) Final all polynomials $f(t)$ of degree 2 or lower (that is, of the form $f(t) = \alpha t^2 + \beta t + \gamma$) whose graph run through the points $(1, 3)$ and $(2, 6)$ such that $f'(1) = 1$ (here $f'(t)$ represents the derivative). Show all work.

Solution. For this problem, we are fitting a curve $f(t)$ to two given points and its derivative $f'(t)$ to a third point. The first point is $(1, 3)$. Evaluating $f(t)$ at this gives us the equation,

$$\alpha + \beta + \gamma = 3.$$

The second point is $(2, 6)$. Evaluating $f(t)$ at this point gives,

$$4\alpha + 2\beta + \gamma = 6.$$

Finally, evaluating $f'(t)$ at the point $(1, 1)$ gives,

$$2\alpha + \beta = 1.$$

Assembling the three equations above into a linear system yields

$$\begin{aligned} \alpha + \beta + \gamma &= 3 \\ 4\alpha + 2\beta + \gamma &= 6 \\ 2\alpha + \beta + 0 &= 1. \end{aligned}$$

We solve the above system using Gauss Jordan elimination. We will also label the equations as done in problem 1.

$$\begin{aligned} -(1) + (2) \to \alpha + \beta + \gamma &= 3 \\ 3\alpha + \beta + 0 &= 3 \\ 2\alpha + \beta + 0 &= 1 \end{aligned}$$

$$\begin{aligned} -(3) + (2) \to \alpha + \beta + \gamma &= 3 \\ \alpha + 0 + 0 &= 2 \\ 2\alpha + \beta + 0 &= 1 \end{aligned}$$

$$\begin{aligned} -2(2) + (3) \to \alpha + \beta + \gamma &= 3 \\ \alpha + 0 + 0 &= 2 \\ 0 + \beta + 0 &= -3 \end{aligned}$$

$$\begin{aligned} -(2 + 3) + (1) \to 0 + 0 + \gamma &= 4 \\ \alpha + 0 + 0 &= 2 \\ 0 + \beta + 0 &= -3. \end{aligned}$$

Therefore,

$$\alpha = 2 \quad \beta = -3 \quad \gamma = 4,$$

and the solution is,

$$f(t) = 2t^2 - 3t + 4.$$

□