We check the 2 conditions for a set to be a subspace

Quiz 4 Solution Guide

Math 4242; Spring 2018; Quiz 4: 15 minutes to complete.

Check:

(1) (10 points) Let X and Y be two subspaces of a vector space V.

(a). (6 pts) Prove that the intersection $X \cap Y$ is also a subspace of V.

① Take $\vec{x} \in X \cap Y$ and $\alpha \in \mathbb{F}$ $(= \mathbb{R} \text{ or } \mathbb{C})$

Then $\vec{x} \in X$ and $\vec{x} \in Y$ so $\alpha\vec{x} \in X$, $\alpha\vec{x} \in Y$

Because X & Y are subspaces. Hence $\alpha\vec{x} \in X \cap Y$

② If $\vec{x}, \vec{y} \in X \cap Y$ then $\vec{x}, \vec{y} \in X$ and $\vec{x}, \vec{y} \in Y$ hence $\vec{x} + \vec{y} \in X$ and $\vec{x} + \vec{y} \in Y$ (Since X, Y are both subspaces.)

Hence $\vec{x} + \vec{y} \in X \cap Y$ as desired.

(b). (4 pts) Show that the union $X \cup Y$ need not be a subspace.

Take $X \subseteq \mathbb{R}^2$, $X = \operatorname{span}\left\{\begin{pmatrix}1\\0\end{pmatrix}\right\}$

$Y \subseteq \mathbb{R}^2$, $Y = \operatorname{span}\left\{\begin{pmatrix}0\\1\end{pmatrix}\right\}$

Then $\begin{pmatrix}1\\0\end{pmatrix} \in X \cup Y$ and $\begin{pmatrix}0\\1\end{pmatrix} \in X \cup Y$

But $\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} \notin X \cup Y$

So it is not a subspace!

I have given an example of how one of the conditions for a subspace does not hold!