

# Solution Guide Quiz 6

**Math 4242; Spring 2018; Quiz: 15 minutes to complete.**  
**No books, no notes**  
**Monday, March 26, 2018**

(1) (15 points) Let  $T$  be the operator on  $R^2$  defined by

$$T(x, y) = (5x - 6y, 3x - 4y)$$

and let  $\vec{v}_0$  be the fixed vector,

$$\vec{v}_0 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

Suppose we have a basis of  $R^2$  given by,

$$B = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$$

Determine  $[T]_B$  and  $[\vec{v}_0]_B$ . You must show your work, and in addition to showing your work you must explain briefly why your answers are what you claim they are. (For example, if you simply write down an answer for  $[\vec{v}_0]$  without writing down something about why this is the answer you will get no credit for that answer.)

$[\vec{v}_0]_B = ??$  : We see by inspection that

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Hence  $[\vec{v}_0]_B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$[T]_B = ??$  : Check that  $T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -1 \\ -1 \end{pmatrix} = -1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 0 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

So  $[T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)]_B = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Also,  $T\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 2 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

So  $[T\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right)]_B = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ . Hence  $[T]_B = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$