Math 4242; Spring 2018; Quiz 11: 10 minutes to complete.
Monday, April 23, 2018

Important: two pages to this quiz! 10 points total.

1). (5 points)

Using Gaussian Elimination to reduce A to an upper triangular matrix, evaluate $\det(A)$ for the following matrix (show all your work),

$$A = \begin{pmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{pmatrix}.$$

Solution. We will use the convention $aR_i + R_j$ to mean multiply Row i by a and add that to Row j. The result is then put in Row j.

$$-R_2 + R_3 \rightarrow \begin{pmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 \\ 1 & 1 & 1 & 2 \end{pmatrix}$$

$$-R_2 + R_4 \rightarrow \begin{pmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$$\begin{matrix} \text{Swap}(R_1, R_2) \rightarrow \\ \text{Swap}(R_3, R_4) \rightarrow \end{matrix} \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 7 & 5 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix}.$$

Then the determinant of the matrix is the product of the diagonal elements. Since we swapped two rows, we must multiply the product by (-1) twice. The answer is then,

$$(-1)(-1)(1)(7)(-1)(2) = 14$$

$$\rightarrow \boxed{14}$$

□

2) (5 points) Use cofactor expansion to evaluate the determinant of the following matrix (show all work),

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix}.$$

Solution. We will expand along the top row. The determinant is then,

$$\begin{aligned} \left| \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix} \right| &= (1) \left| \begin{pmatrix} 5 & 6 \\ 8 & 10 \end{pmatrix} \right| - (2) \left| \begin{pmatrix} 4 & 6 \\ 7 & 10 \end{pmatrix} \right| + (3) \left| \begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix} \right| \\ &= 1(50 - 48) - 2(40 - 42) + (32 - 35) \\ &= 2 - (-4) + (-9) \\ &= -3. \end{aligned}$$

So the answer is,

$$\boxed{\left| \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{pmatrix} \right| = -3}$$

□