Key.

Math 4242; Spring 2018; Quiz 7: 10 minutes to complete.
Monday, April 2, 2018

1). (10 points) For $\vec{x} \in R^n$ (any $n \geq 1$) explain why,

$$\|\vec{x}\|_1 \geq \|\vec{x}\|_2 \geq \|\vec{x}\|_\infty$$

(1) (2)

To see (2) Above:

If $\vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ then

$$\|\vec{x}\|_\infty^2 = \max_{i=1,\dots,n} |x_i|^2 \leq |x_1|^2 + \cdots + |x_n|^2$$

The $\max_{i=1,\dots,n} |x_i|^2$ is one of these.

$$= \|x\|_2^2$$

Hence $\|x\|_\infty \leq \|x\|_2$.

To see (1): $\|\vec{x}\|_2^2 = |x_1|^2 + |x_2|^2 + \cdots + |x_n|^2$

and $\|\vec{x}\|_1^2 = (|x_1| + \cdots + |x_n|)^2 = |x_1|^2 + |x_2|^2 + \cdots + |x_n|^2 + \sum_{i<j} 2|x_i| \cdot |x_j|$

≥ 0

$$\geq |x_1|^2 + \cdots + |x_n|^2$$

$$= \|x\|_2^2$$

Hence $\|\vec{x}\|_1 \geq \|\vec{x}\|_2$.

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