Key

Math 4242; Spring 2018; Quiz 8: 12 minutes to complete.
Monday, April [redacted], 2018
9.

Important: two pages to this quiz! Make sure to turn the sheet over and continue your work!

1). (10 points) If we use the notation,

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \qquad \vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix},$$

determine which of the following is an inner product on R^3. If it is an inner product, explain why. If it is NOT an inner product, explain why it is not.

(a). $< \vec{x}, \vec{y} > = x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2$.

This is <u>not</u> an inner product.

for example, for $\lambda \in \mathbb{R}$, $\lambda \neq 0$

$$\langle \lambda \vec{x}, \vec{y} \rangle = \lambda^2 \langle \vec{x}, \vec{y} \rangle \neq \lambda \langle \vec{x}, \vec{y} \rangle$$

(b). $< \vec{x}, \vec{y} > = x_1 y_1 + x_3 y_3$. This is <u>not</u> an inner product.

for example, if $\vec{x} = \vec{y} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Then $\langle \vec{x}, \vec{x} \rangle = 0$ but $\vec{x} \neq \vec{0}$.

2) (10 points) Using the standard inner product, determine the Fourier expansion of $\vec{x}$ with respect to the basis B where we have,

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad B = \left\{ \underbrace{\frac{1}{\sqrt{6}}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}}_{\vec{v}_1}, \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}_{\vec{v}_2}, \underbrace{\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\vec{v}_3} \right\}$$

We have

$$\langle \vec{x}_1, \vec{v}_1 \rangle = \frac{1}{\sqrt{6}} \cdot (1+2+2) = \frac{5}{\sqrt{6}}$$

$$\langle \vec{x}_1, \vec{v}_2 \rangle = \frac{1}{\sqrt{2}} (-1+2) = \frac{1}{\sqrt{2}}$$

$$\langle \vec{x}_1, \vec{v}_3 \rangle = \frac{1}{\sqrt{3}} (1+2-1) = \frac{2}{\sqrt{3}}$$

So fourier expansion is

$$\vec{x} = \frac{5}{\sqrt{6}} \vec{v}_1 + \frac{1}{\sqrt{2}} \vec{v}_2 + \frac{2}{\sqrt{3}} \vec{v}_3$$

$$= \frac{5}{6}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$