The Natural Numbers

In this course we will explore properties of the natural numbers:

\[ \mathbb{N} = \{1, 2, 3, \ldots\}. \]

1. Complete this definition: “A natural number \( n \) is odd if . . . ”
2. Complete this definition: “A natural number \( n \) is composite (that is, not prime) if . . . ”
3. Write a true statement that starts with the words “There is a natural number . . . ” and contains the words “odd” and “prime”.
4. Write a false statement that starts with the words “There is a natural number . . . ” and contains the words “odd” and “prime”.

We count things with the natural numbers – sea shells, home runs, the terms in a sequence, subdivisions of an interval, etc.

The Real Numbers and Some Calculus

In this course we will also explore properties of the real numbers \( \mathbb{R} \) and the behavior of functions generally and of sequences, which are simply functions for which the domain is the set \( \mathbb{N} \) of natural numbers. By doing this we will gain a better understanding of the calculus that we learn in high school or in the first year of college.

This exercise concerns the expression

\[ \int_{0}^{1} (2x + 3) \, dx. \]

1. Find the value of the definite integral above using a geometric interpretation; that is, draw a graph of the integrand, and find the area of a polygon.
2. Find the value of the definite integral above using the fundamental theorem of calculus.
3. State the fundamental theorem of calculus. There are two parts! Which one did you use in #2?
4. State the definition of the derivative \( f'(a) \) of a function \( f \) at \( x = a \). Your statement should contain a limit. What is the input that is changing in the limit?
5. State a definition of the definite integral

\[ \int_{a}^{b} f(x) \, dx \]

of a function \( f \) on a closed interval \([a, b]\). Your statement should contain a limit. What is the input that is changing in the limit?
6. Find the value of the definite integral above, using the definition of the definite integral directly, by dividing \([0, 1]\) into \( n \) equal-width subintervals, and evaluating the function at the right endpoint of each subinterval.

A main goal of this course is to understand precisely what it means for the limits you wrote above to exist. That is, what is the precise definition of convergence, for sequences and for functions of a real variable?