Part A

1. Let $\langle a_n \rangle$ be a sequence. Prove that $a_n \longrightarrow 0$ if, and only if $|a_n| \longrightarrow 0$.

2. Let $a_n = \frac{\cos(n\pi)}{1+n^{3/2}}$. (Note: In the phrase "Let $a_n = \ldots$ " it is implied that the assignment holds for each $n \in \mathbb{N}$).
   (a) Find an equation for a_n which does not involve cosines.
   (b) What is $\lim_{n\to\infty} a_n$? Give reasons for your answer.

3. Let $a_n = \sin\left(\frac{\pi n-1}{1-2n} \ln(\frac{en^2-e}{(n+1)^2})\right)$. Does $\langle a_n \rangle$ converge? If so, find $L = \lim_{n\to\infty} a_n$. Give reasons for your answer.

4. Let $a_n = \frac{3^n}{n!}$
   (a) Show that for all n, $a_n \leq \frac{9}{2}\frac{3}{n}$.
   (b) Does $\langle a_n \rangle$ converge? If so, find $L = \lim_{n\to\infty} a_n$.

Part B

5. Let $a_n = \frac{n^2-1}{2n^2+1}$.
   (a) Show that $\langle a_n \rangle$ converges. What is $L = \lim_{n\to\infty} a_n$?
   (b) Find a value n_0 such that $|a_{n_0} - L| < \frac{1}{10^2}$. Show your work.

6. Determine if the following are true or false. If true, prove it. If false, give a counterexample.
   (a) If $\langle a_n + b_n \rangle$ is divergent, then either $\langle a_n \rangle$ is divergent or $\langle b_n \rangle$ is divergent.
   (b) If $\langle a_n \rangle$ and $\langle a_n + b_n \rangle$ are convergent, then $\langle b_n \rangle$ is convergent.
   (c) If $\langle a_n \rangle$ and $\langle a_n b_n \rangle$ are convergent, then $\langle b_n \rangle$ is convergent.

7. Let $s_n = \sum_{i=0}^{n} \frac{1}{2^i} = 1 + \frac{1}{2} + \ldots + \frac{1}{2^n}$. Prove that $\langle s_n \rangle$ converges. What is $S = \lim_{n\to\infty} s_n$?