Math 3283, Homework 1

(due Monday 2/1)

Directions: On each page, write your name, your section TA's name, and your section number

To allow comments, have at most 2 problems on each side of a page.

State each problem before writing the solution.

Please write clearly and legibly; sloppy or unreadable work will be penalized.

Part A

1. Exercise 1.3, p. 5.
2. Exercise 2.12, p. 11.
3. Prove, using truth tables, that $\sim(P \wedge Q \wedge R) \equiv (\sim P) \vee (\sim Q) \vee (\sim R)$.

Part B

1. Prove that for all $n \in \mathbb{N}$, n^3 is even if and only if n is even.
2. Prove that every positive real number is greater than the cube of some other positive real number.
3. Let $P(x)$ be the statement "$f(w)$ is continuous at x", and let $Q(x)$ be the statement "$f(w)$ is differentiable at x". (Here, f is a function $f: \mathbb{R} \to \mathbb{R}$, and x is a real number.) Use logical notation (including quantifiers) to rewrite the following statements:

   (a) $f(w)$ is differentiable on $(0,1)$ and continuous on $[0,1]$;

   (b) There is a real number x where f is continuous but not differentiable;

   (c) $f(w)$ is not continuous for any negative real number;

   (d) If x is a real number and $f(w)$ is differentiable at x, then $f(w)$ is continuous at x.

4. Let p be any prime number. Prove that the statement "for all $n \in \mathbb{N}$, $n^2 + n + p$ is prime" is false.