1. Determine if the following series converge or diverge. Carefully show all your reasoning.

   a) \[ \sum_{n=1}^{\infty} \frac{1}{3^n - 2^n} \]
   b) \[ \sum_{n=2}^{\infty} \frac{1}{n^2 - \sqrt{n}} \]
   c) \[ \sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^4 + 1}} \]

2. Suppose \[ \sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n \] are positive series such that \( \forall n \in \mathbb{N}, 0 < a_{2n} \leq b_{2n} \).

   (1) If \( \sum b_n \) converges, does \( \sum a_n \) converge?
   (2) If \( \sum a_n \) diverges, does \( \sum b_n \) diverge?

3. Determine if the series \[ \sum_{n=1}^{\infty} \int_{\frac{n}{2}}^{\frac{2n}{3}} \frac{dx}{x} \] converges or diverges. Carefully show all steps in your reasoning.

   Hint: Rewrite the series using terms \[ \int_{\frac{n}{2}}^{\frac{2n}{3}} \frac{dx}{x} \]

4. a) Prove that if the positive series \[ \sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n \] converge, then \[ \sum_{n=1}^{\infty} a_n b_n \] converges.

   b) If \[ \sum_{n=1}^{\infty} a_n \] positive series that converges and \( m \in \mathbb{N} \), prove that \[ \sum_{n=1}^{\infty} a_n^m \] converges.