1. Let \( A, B \) be finite sets where \( \#A = m \) and \( \#B = n \), let \( f: A \rightarrow B \) be a function.
1. Prove that if \( f \) is 1-to-1, then \( m \leq n \).
2. Prove that if \( f \) is a bijection, then \( m = n \).

2. Let \( A, B \) be finite sets and \( F(A,B) \) the set of all functions \( f: A \rightarrow B \).
1. Suppose \( A = \{a_1, a_2, a_3\} \) and \( B = \{1, 2, 3\} \), find \( \#F(A,B) \). Show your reasoning.
2. Suppose \( A = \{a_1, a_2, a_3, a_4\} \) and \( B = \{1, 2, 3\} \), find \( \#F(A,B) \). Show your reasoning.
3. Suppose \( \#A = m \) and \( \#B = n \), find \( \#F(A,B) \). Prove that your reasoning is correct.

3. Consider the function \( f: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \) defined by
\[
f(n,m) = \frac{1}{2} (n+m-2)(n+m-1) + m
\]
1. Compute \( f(1,1), f(2,1), f(1,2), f(1,3), f(2,2) \) and \( f(3,1) \).
2. Show that \( f(n,m) = f(n', m') \) implies \( (n,m) = (n', m') \) in the following cases: a) \( n + m = n' + m' \), b) \( n + m \) not equal to \( n' + m' \), but \( n = n' \) c) \( n + m \) not equal to \( n' + m' \), but \( m = m' \).
3. Find pairs \( (n,m) \) such that a) \( f(n,m) = 13 \), b) \( f(n,m) = 19 \), c) \( f(n,m) = 28 \).
4. It can be shown that \( f \) is onto. How is \( f \) related to the bijection \( g: \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N} \) described in the lecture notes?