Part A

1. Determine if the following sets are bounded above. If so, find the least upper bound. Are they bounded below? If so, find the greatest lower bound.
   (a) $\{x \in \mathbb{R} | x < 0 \text{ and } x^2 + 2x > 0\}$
   (b) $\{\frac{1}{2}, -\frac{1}{2}, \frac{2}{3}, -\frac{2}{3}, \frac{3}{4}, -\frac{3}{4}, \ldots\}$
   (c) $\{1 - .9, 1 - .99, 1 - .999, 1 - .9999, \ldots\}$
2. Let $A = \{\sin(x) | \frac{\pi}{4} \leq x \leq \frac{7\pi}{4}\}$
   (a) Show that A is bounded above and below.
   (b) Find $\sup(A)$ and $\inf(A)$. Show your work.
   (c) Is $\sup(A)$ a member of A? Is $\inf(A)$ a member of A?
3. Let A be a non-empty subset of $\mathbb{R}$ which is bounded above.
   (a) Show that $-A$ is bounded below, where $-A = \{-a | a \in A\}$.
   (b) Let $w = \sup(A)$. Prove that $\inf(-A) = -w$.
4. Suppose $A \subset \mathbb{R}$ and $B \subset A$ is non-empty. What can you conclude about $\inf(A)$ and $\inf(B)$? Prove your result.

Part B

5. Find the least integer k such that $4^k > k^4$ for all $n \geq k$. Prove by induction that your answer is correct.
6. Consider the set $A = \{2.1, -2.3, 2.11, -2.33, 2.111, -2.333, \ldots\}$.
   (a) Write a general expression for the $2n^{th}$ and $(2n+1)^{st}$ elements.
   (b) Find the set B of upper bounds of A and the set C of lower bounds of A.
   (c) What is $\inf(A)$? Prove your answer is correct.
   (d) What is $\sup(A)$? Prove your answer is correct.
   (e) Find $r \in \mathbb{R}$ such that $r \notin B \cup C$.
7. Let $a > 0$. What is $\inf\{\frac{a}{n} | n \in \mathbb{N}\}$? Prove that your answer is correct.