

## HOMework 1 (DUE: 11:15 AM, SEP 19 WED)

1. Do Exercise 2.B.1–6. in [Pin10, pp. 23–24]. You do not need to justify your answers.
2. Do Exercise 3.C.1–3. in [Pin10, pp. 30–31]. Here you *do* need to justify your answers.
3. [Fra02, Exercise 4.19] Let  $S := \mathbb{R} - \{-1\}$  with the operation

$$* : S \times S \rightarrow S : (a, b) \mapsto a + b + ab.$$

- a. Show that  $(S, *)$  is a group.
  - b. Find the solution of the equation  $2 * x * 3 = 7$  in  $S$ .
4. [Sar08, Exercise 3.11] Let  $(G, *)$  be a group such that  $x^2 = e$  for all  $x \in G$ . Show that  $(G, *)$  is abelian, i.e.  $*$  is commutative.
  5. [Sar08, Exercise 3.14] Let  $G$  be a nonempty set with an associative operation  $* : G \times G \rightarrow G$ . If  $\forall a, b \in G, \exists x, y \in G$  such that  $a * x = y * a = b$ , then show that  $(G, *)$  is a group.

### REFERENCES

- [Fra02] Fraleigh, J. B., *A First Course in Abstract Algebra*, 7th ed., Pearson, 2002.  
[Pin10] Pinter, C. C., *A Book of Abstract Algebra*, 2nd ed., Dover Publications, 2010.  
[Sar08] Saracino, D., *Abstract Algebra: A First Course*, 2nd ed., Waveland Press, 2008.