1. Exercise 14.1] and Exercise 11.21–25] Find, up to isomorphism, all abelian groups of order:

(a) 32  (b) 48  (c) 72  (d) 84  (e) 450  (f) 720  (g) 1089

Also, find the maximum possible order for some element in each group. You do not need to justify your answer.

2. Exercise 11.29] Here you do not need to justify your answer.

(a) Let $p$ be a prime number. Find the number of abelian groups, up to isomorphism, of order $p^n$ when $n = 2, 3, 4, 5, 6, 7, 8$.

(b) Let $p, q, r$ be distinct prime numbers. Find the number of abelian groups, up to isomorphism, of order:

i. $p^3q^4r^7$  
ii. $(qr)^7$  
iii. $p^5q^4r^3$.

3. Exercise 11.10] Find all subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. You do not need to justify your answer.


(a) Are the groups $\mathbb{Z}_2 \times \mathbb{Z}_{12}$ and $\mathbb{Z}_4 \times \mathbb{Z}_6$ isomorphic? Why or why not?

(b) Are the groups $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_{24}$ and $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{40}$ isomorphic? Why or why not?

(c) Are the groups $\mathbb{Z}_4 \times \mathbb{Z}_{18} \times \mathbb{Z}_{15}$ and $\mathbb{Z}_3 \times \mathbb{Z}_{36} \times \mathbb{Z}_{10}$ isomorphic? Why or why not?

**References**
