

HOMEWORK 7 (BONUS!) (DUE: 11:15 AM, OCT 31 WED)

1. [Sar08, Exercise 14.1] and [Fra02, Exercise 11.21–25] Find, up to isomorphism, all abelian groups of order:

(a) 32 (b) 48 (c) 72 (d) 84 (e) 450 (f) 720 (g) 1089

Also, find the maximum possible order for some element in each group. You do not need to justify your answer.

2. [Fra02, 11.29] Here you do not need to justify your answer.
- (a) Let p be a prime number. Find the number of abelian groups, up to isomorphism, of order p^n when $n = 2, 3, 4, 5, 6, 7, 8$.
- (b) Let p, q, r be distinct prime numbers. Find the number of abelian groups, up to isomorphism, of order:

i. $p^3q^4r^7$ ii. $(qr)^7$ iii. $p^5q^4r^3$.

3. [Fra02, Exercise 11.10] Find all subgroups of $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. You do not need to justify your answer.
4. [Fra02, Exercise 11.15–20] Here you *need* to justify your answer.
- (a) Are the groups $\mathbb{Z}_2 \times \mathbb{Z}_{12}$ and $\mathbb{Z}_4 \times \mathbb{Z}_6$ isomorphic? Why or why not?
- (b) Are the groups $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_{24}$ and $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{40}$ isomorphic? Why or why not?
- (c) Are the groups $\mathbb{Z}_4 \times \mathbb{Z}_{18} \times \mathbb{Z}_{15}$ and $\mathbb{Z}_3 \times \mathbb{Z}_{36} \times \mathbb{Z}_{10}$ isomorphic? Why or why not?

REFERENCES

- [Fra02] Fraleigh, J. B., *A First Course in Abstract Algebra*, 7th ed., Pearson, 2002.
[Sar08] Saracino, D., *Abstract Algebra: A First Course*, 2nd ed., Waveland Press, 2008.