

HOMEWORK 8 (DUE: 11:15 AM, NOV 7 WED)

1. [Fra02, Exercise 16.12] Let X be a G -set and let $Y \subset X$. Let

$$G_Y := \{g \in G \mid gy = y \text{ for all } y \in Y\}.$$

Show that G_Y is a subgroup of G .

2. [Fra02, Exercise 16.13] Suppose that $G = \mathbb{R}$ acts on \mathbb{R}^2 in a way that $x \in \mathbb{R}$ acts by the counterclockwise rotation of \mathbb{R}^2 about the origin through x radians.

(a) Show that this action is well-defined.

(b) Describe each orbit of \mathbb{R} in \mathbb{R}^2 geometrically.

(c) For each $p \in \mathbb{R}^2$, what is $\text{Stab}_{\mathbb{R}}(p)$?

3. The edges of a regular n -gon are to be painted with r different colors. If it is allowed to use each color several times, find the number of ways for such paintings up to rotation if

(a) $(n, r) = (6, 2)$,

(b) $(n, r) = (3, 4)$.

Briefly justify your answers.

4. Do the problem 3 if “up to rotation” is replaced by “up to rotation and reflection”.

REFERENCES

[Fra02] Fraleigh, J. B., *A First Course in Abstract Algebra*, 7th ed., Pearson, 2002.