PROBLEM SET 10 (DUE: APR 24, WED)

1. Suppose that we are given an n by m matrix A whose column vectors are linearly independent an n by n symmetric positive definite matrix C. Set $P = A(A^TCA)^{-1}A^TC$.
   (a) Show that A^TCA is invertible. (Thus, P is well-defined.)
   (b) Prove that $P^2 = P$ and $C^{-1}P^TC = P$.
   (c) Prove that $\operatorname{im} P = \operatorname{im} A$.
   (d) Prove that $P = I$ if A is a square matrix.
   (e) Prove that $(v - Pv) \perp \operatorname{im} A$ for any $v \in \mathbb{R}^n$ with respect to the inner product $\langle x, y \rangle = x^TCy$.
2. Do Exercise 4.4.11. in [OS18].
3. Do Exercise 5.2.1. in [OS18].
4. Do Exercise 5.2.9. in [OS18].
5. Do Exercise 5.3.1. in [OS18].
6. Do Exercise 5.3.2. in [OS18].

— More Exercises Suggestions (these are not a part of homework): 4.4.9, 4.4.10, 4.4.15, 4.4.22, 4.4.28, 4.4.29, 4.4.31, 5.2.3, 5.2.4, 5.2.8, 5.2.11, 5.3.4, 5.3.9, 5.3.14, 5.3.15

References

[OS18] Olver, P. J. and Shakiban, C., *Applied Linear Algebra*, 2nd ed., Undergraduate Texts in Mathematics, Springer, 2018.