When working through this worksheet (and all worksheets), make sure that you are reading carefully. It is important that you re-read things and ask questions if what you are reading is not resonating with you.

**Tangent Problems**

Recall our discussion about the development of calculus.

Calculus developed out of a need to model the dynamic movement of fluid as well as a desire to model the force of gravity. Because of the scientific progress being made in the 17th century, it was desirable to find the rate of change of a function at a single instant.

For example, you know that if you were to drive 50 miles and arrive at your destination in 1 hour, that you were driving an average speed of 50 miles per hour. However, you were not driving that speed during your whole trip. For instance, you probably pulled out of your driveway going somewhere between 5 and 10 miles per hour, not 50! But say you wanted to figure out your speed at one particular instant of your trip...it is calculus that makes this possible.

Let’s now examine a particular example that would have concerned 17th century Europeans somewhat more directly, to get a better idea of how this instantaneous rate of change (or derivative as we will come to call it) is calculated.

**The Velocity Problem**

Consider the following scenario.

Suppose a ball is dropped from the roof of the IDS Tower in Minneapolis, 241 meters above ground. Find the velocity of the ball after 3 seconds.

In the 16th century, Galileo discovered that the distance fallen by any freely falling object is proportional to the square of the time that it has been falling.

If the distance fallen after \( t \) seconds is denoted by \( s(t) \) and measured in meters, then Galileo’s law is expressed by the equation

\[
s(t) = 4.9t^2.
\]

Sketch a graph of this function, labeling the axes.
Note that this function is measuring the distance that an object has fallen as a function of time.

How do you find the average velocity of an object if you know that at time $t_1$ it is at position $s(t_1)$ and at time $t_2$ it is at position $s(t_2)$?

The difficulty with finding the velocity at 3 seconds, is that we are dealing with a single instant of time as opposed to an interval. However, we can approximate the desired quantity by computing the average velocity over the brief time interval $t_1 = 3.0$ and $t_2 = 3.1$.

Compute this velocity.

Again, what you calculated is the average speed at an interval close to $t = 3$ but not the exact velocity at $t = 3$. To get an even more accurate value, complete the following table.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Average Velocity</th>
<th>Time Interval</th>
<th>Average Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \leq t \leq 4$</td>
<td></td>
<td>$2 \leq t \leq 3$</td>
<td></td>
</tr>
<tr>
<td>$3 \leq t \leq 3.1$</td>
<td></td>
<td>$2.9 \leq t \leq 3$</td>
<td></td>
</tr>
<tr>
<td>$3 \leq t \leq 3.05$</td>
<td></td>
<td>$2.95 \leq t \leq 3$</td>
<td></td>
</tr>
<tr>
<td>$3 \leq t \leq 3.01$</td>
<td></td>
<td>$2.99 \leq t \leq 3$</td>
<td></td>
</tr>
<tr>
<td>$3 \leq t \leq 3.001$</td>
<td></td>
<td>$2.999 \leq t \leq 3$</td>
<td></td>
</tr>
</tbody>
</table>

What do you notice about your results?
Refer back to the graph of the $s(t) = 4.9t^2$. How does the chart that you completed relate to this graph? (You may want to mark the function at the intervals that you are discussing.)

As it turns out, the numbers that you were calculating were the slopes of the secant lines that pass through the graph and the indicated time values. Try to sketch a graph modeling this phenomenon.

As your intervals get smaller and smaller (the endpoints get closer to 3), you are getting closer and closer to finding the slope of the line tangent to (touching but not crossing through) the function $s(t)$ at $t = 3$. In other words, you are getting closer and closer to finding the velocity of the ball at 3 seconds after it has been dropped.

What would you estimate this velocity is, based on your chart?

The rest of this chapter will be discussing techniques for more accurately arriving at this value. The main tool that we will be discussing in what follow is the concept of the limit.